Temporal waveform analysis
with convolutional sparse coding models

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13 Apr 2021

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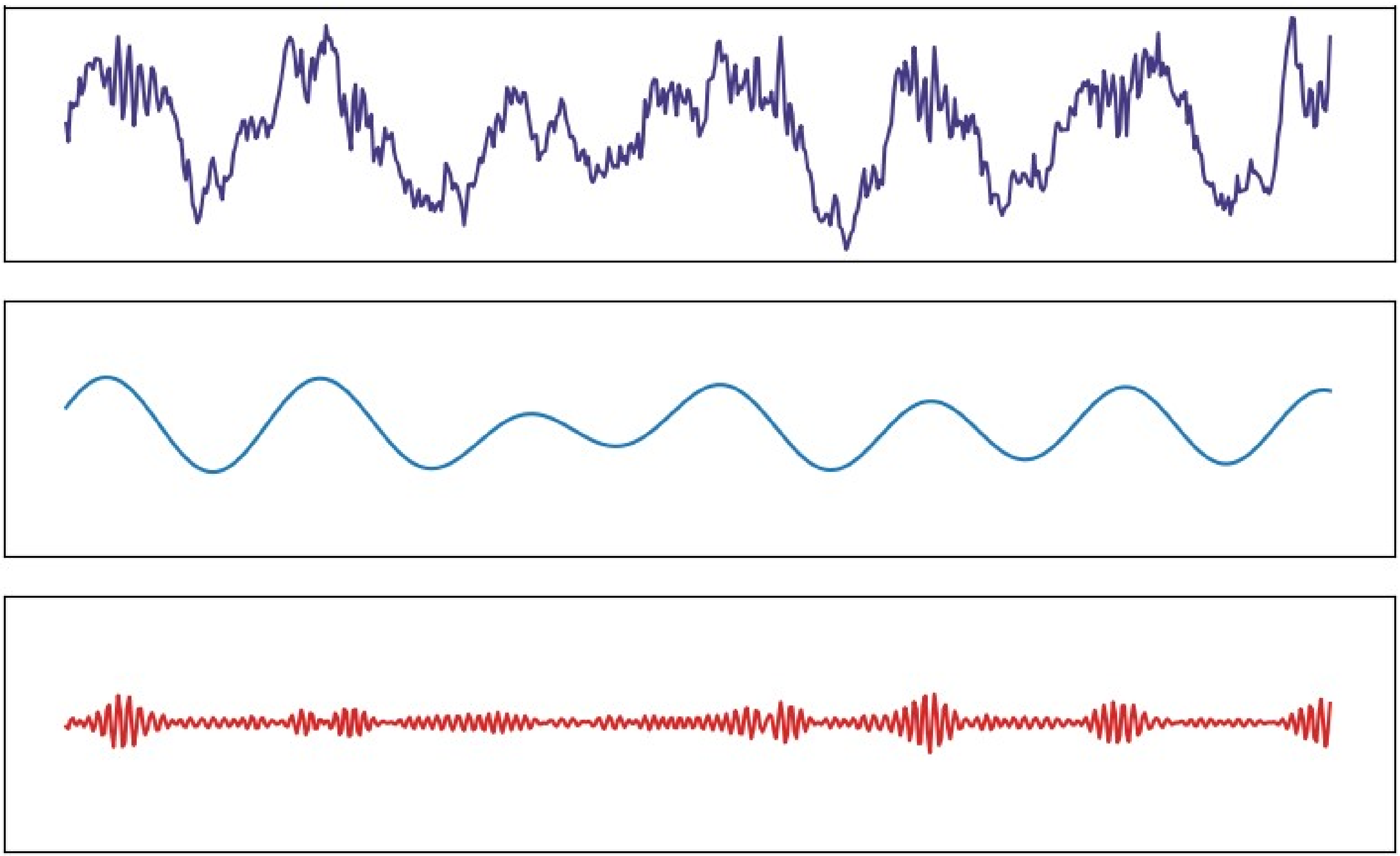
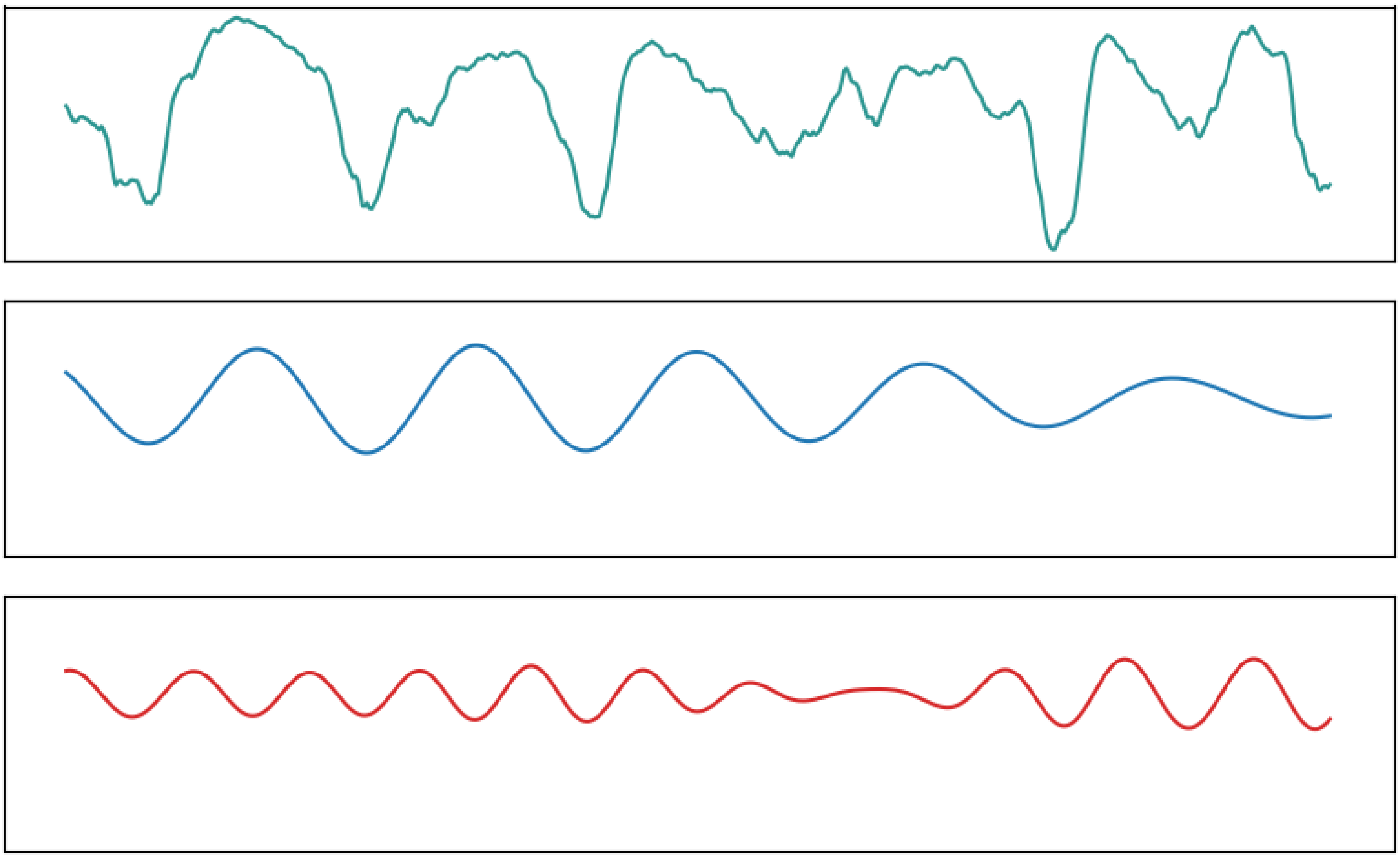
Narrow-band representation?

LFP



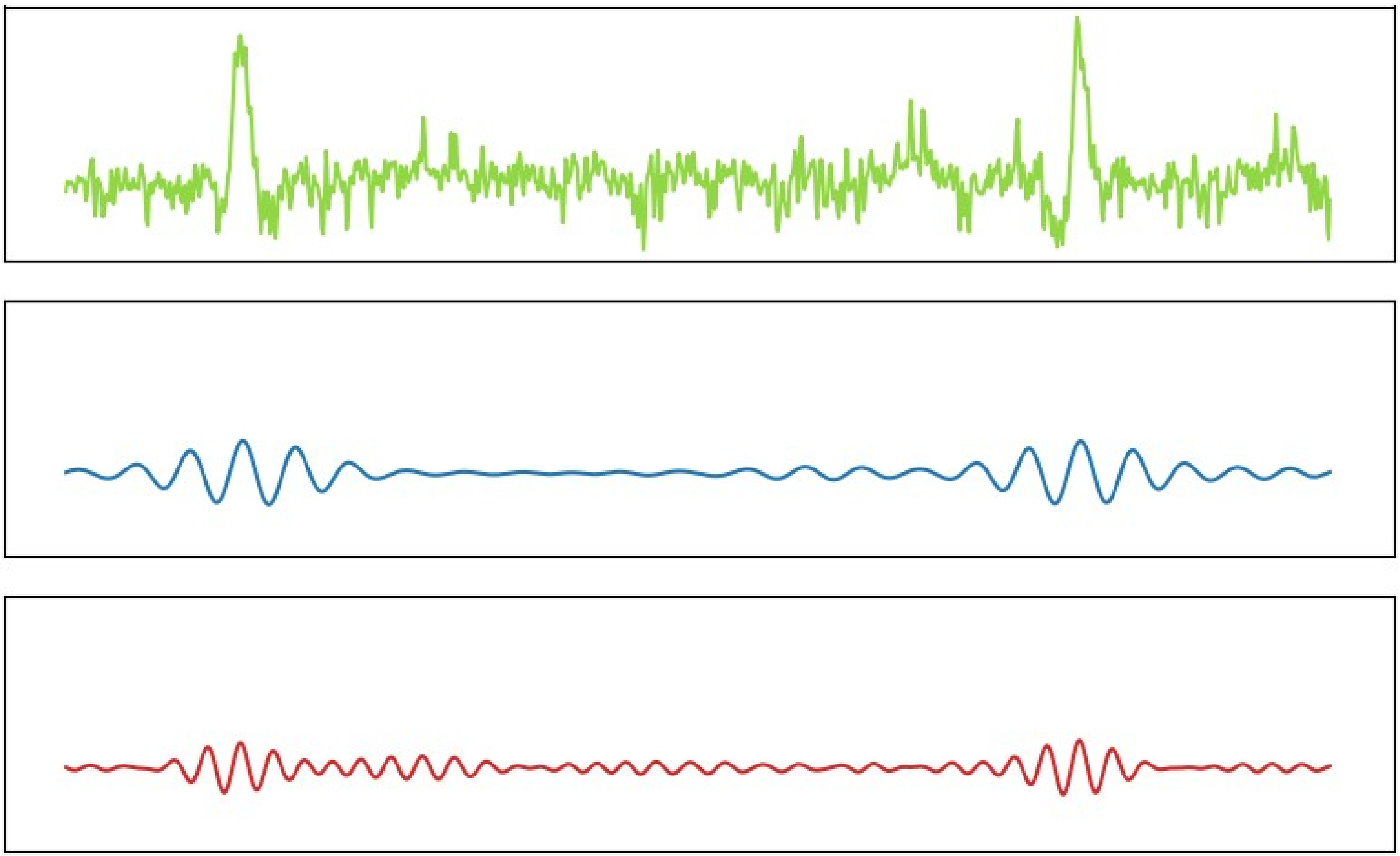
Narrow-band representation?

ECoG



Narrow-band representation?

MEG



Temporal waveform analysis

- Sparse representations: wavelet basis

(Mallat and Zhang, 1993, Candès et al, 2006)

- Sparse coding / dictionary learning

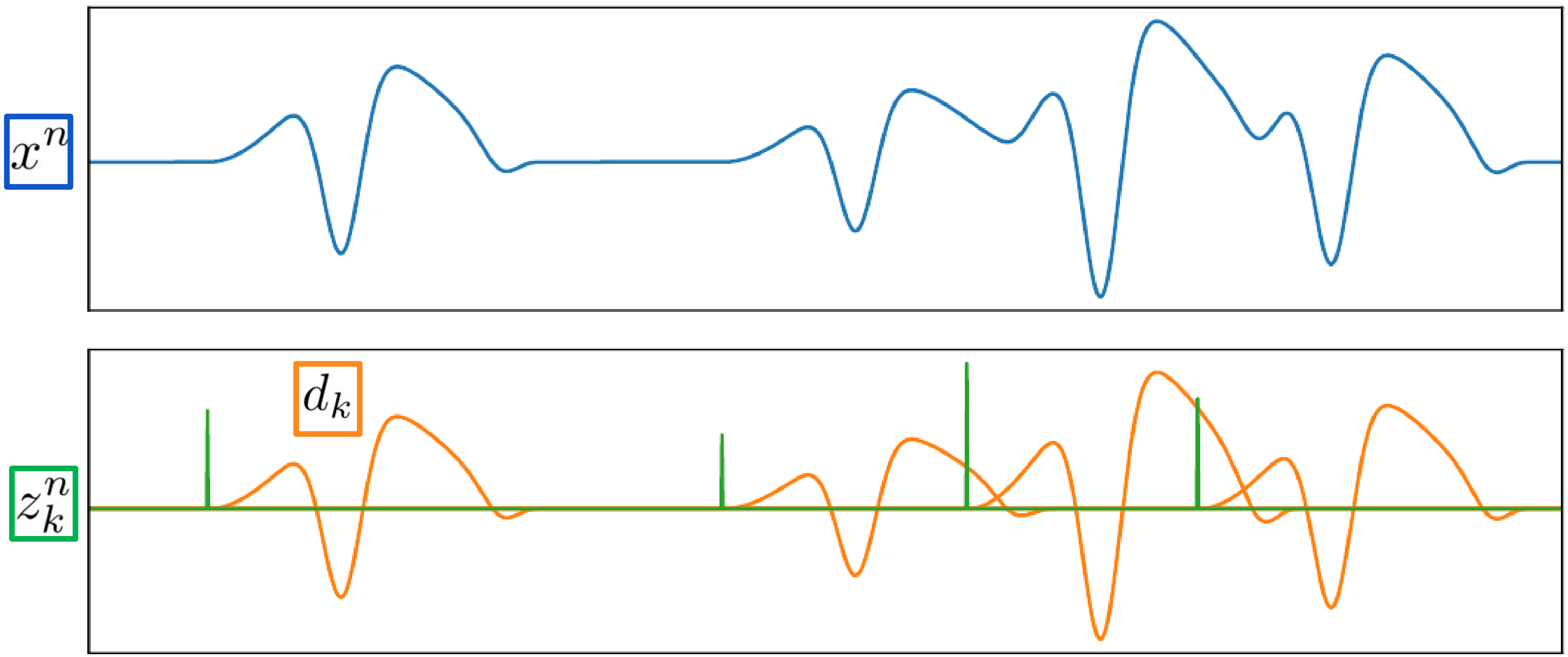
(Olshausen and Field, 1996, Elad and Aharon, 2006)

- Shift-invariant representations

(Lewicki and Sejnowski, 1999, Grosse et al, 2007)

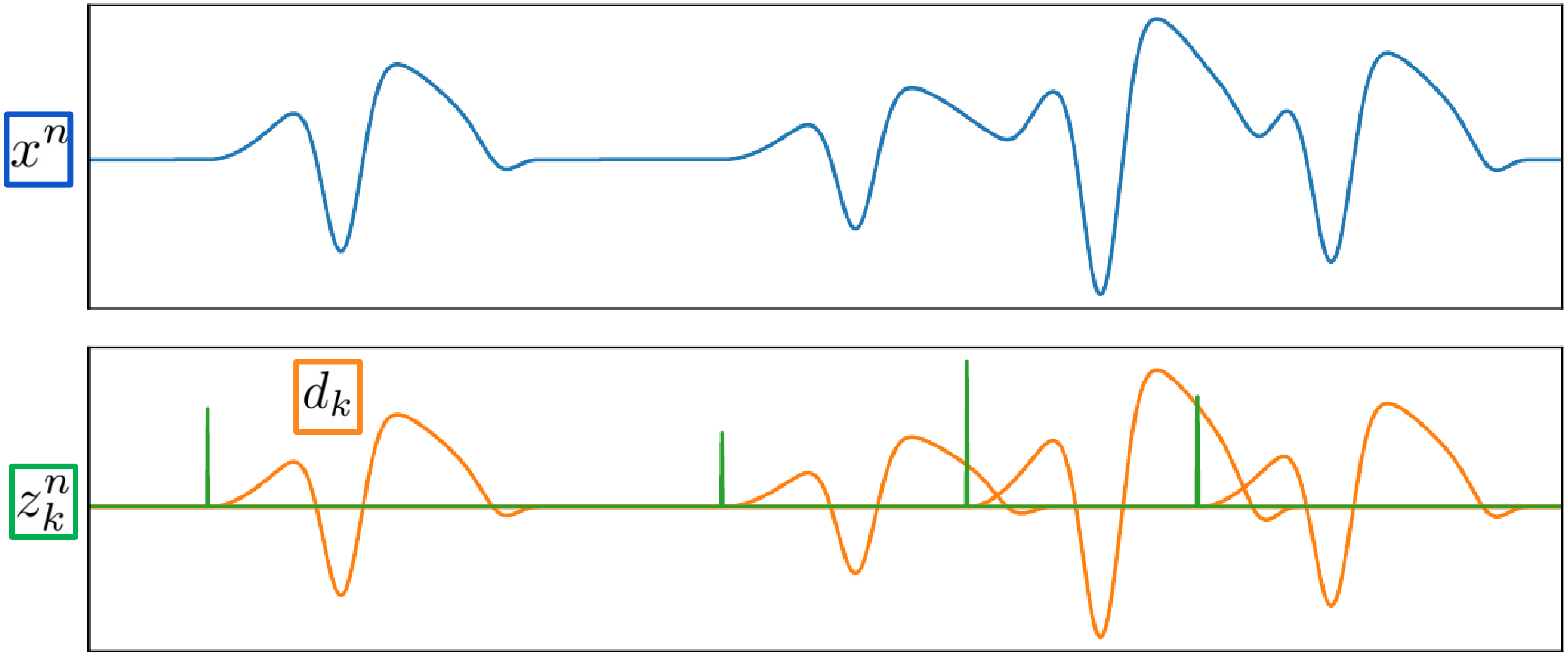
- In neurophysiology:
  - Matching of time-invariant filters (Jost et al, 2006)
  - Multivariate orthogonal matching pursuit (Barthélemy et al, 2012)
  - Matching pursuit and heuristics (Brokmeier and Principe, 2016)
  - Sliding window machine (Gips et al, 2017)
  - Adaptive waveform learning (Hitziger et al, 2017)

Convolutional sparse coding



(Grosse et al, 2007)

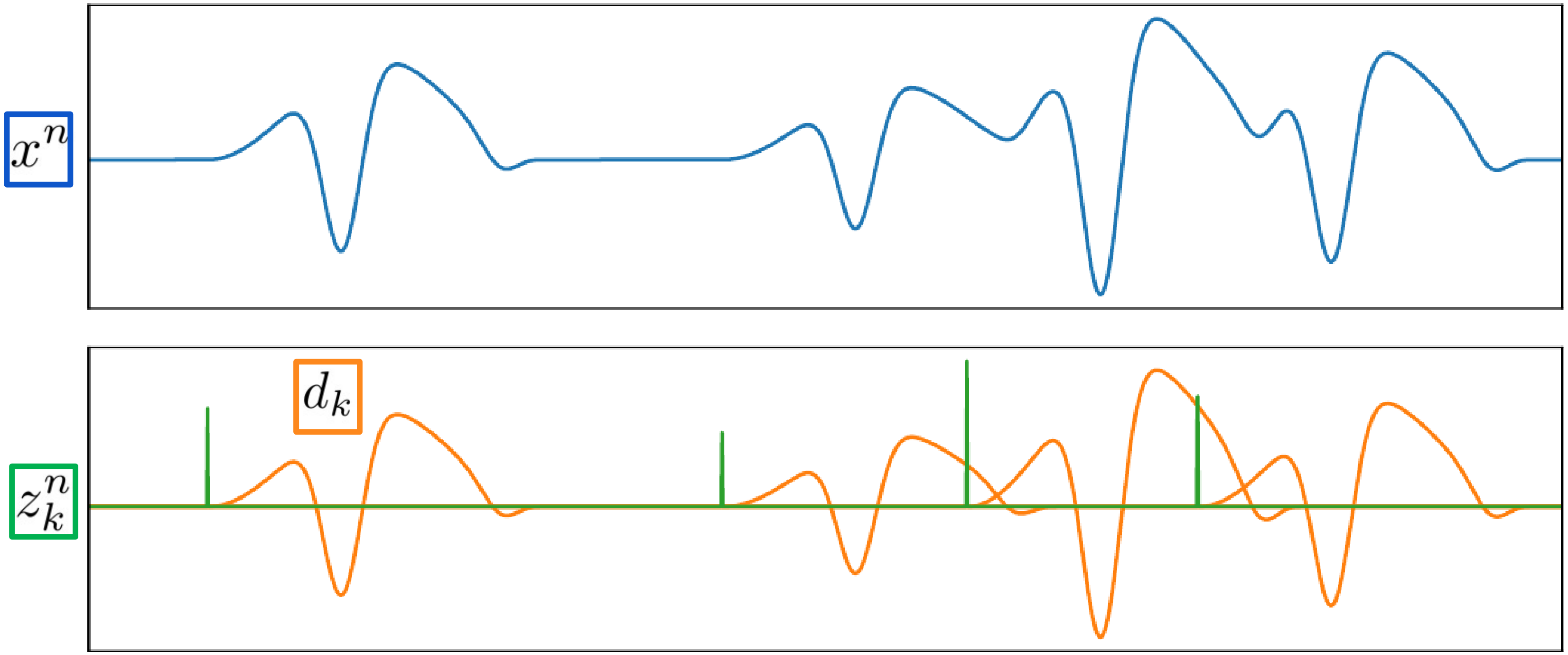
Convolutional sparse coding



$$x^n[t] = \sum_{k=1}^{K} (z_k^n * d_k)[t] + \varepsilon[t]$$

(Grosse et al, 2007)

Convolutional sparse coding

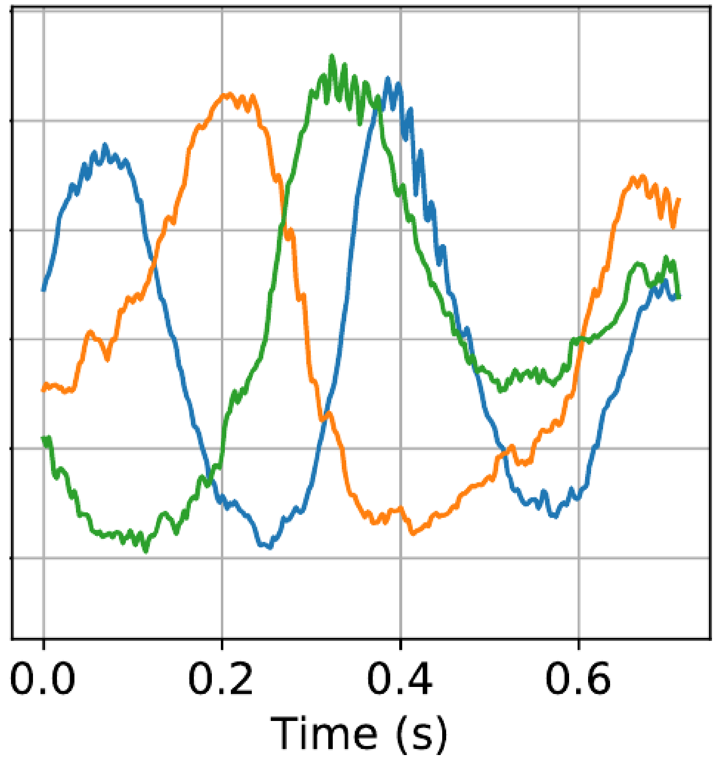


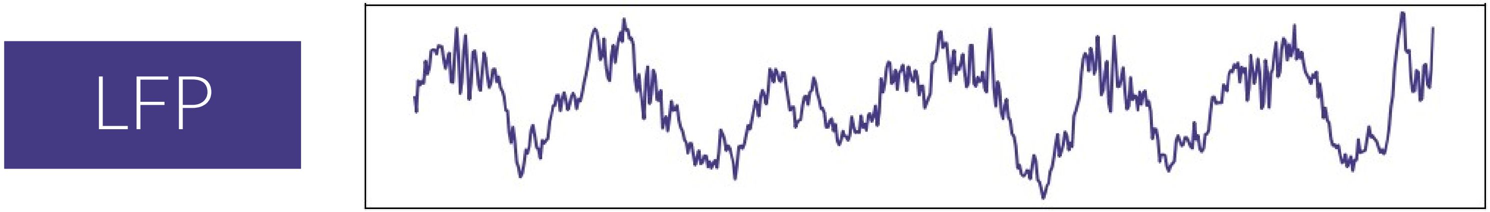
$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$

$$\text{s.t.} \quad \|d_k\|_2^2 \le 1 \text{ and } z_k^n \ge 0.$$

(Grosse et al, 2007)

Learned atoms





First challenge: optimization speed

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \| z_k^n \|_1,$$

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Block-coordinate descent

- Z-step
- D-step

First challenge: optimization speed

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Block-coordinate descent

- Z-step
  - GCD (Kavukcuoglu et al, 2010)
  - FISTA (Chalasani et al, 2013)
  - ADMM (Bristow et al, 2013)
  - ADMM + FFT (Wohlberg, 2016)
  - L-BFGS (Jas et al, 2017)
  - LGCD (Dupré la Tour et al, 2018)
- D-step

First challenge: optimization speed

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$

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Block-coordinate descent

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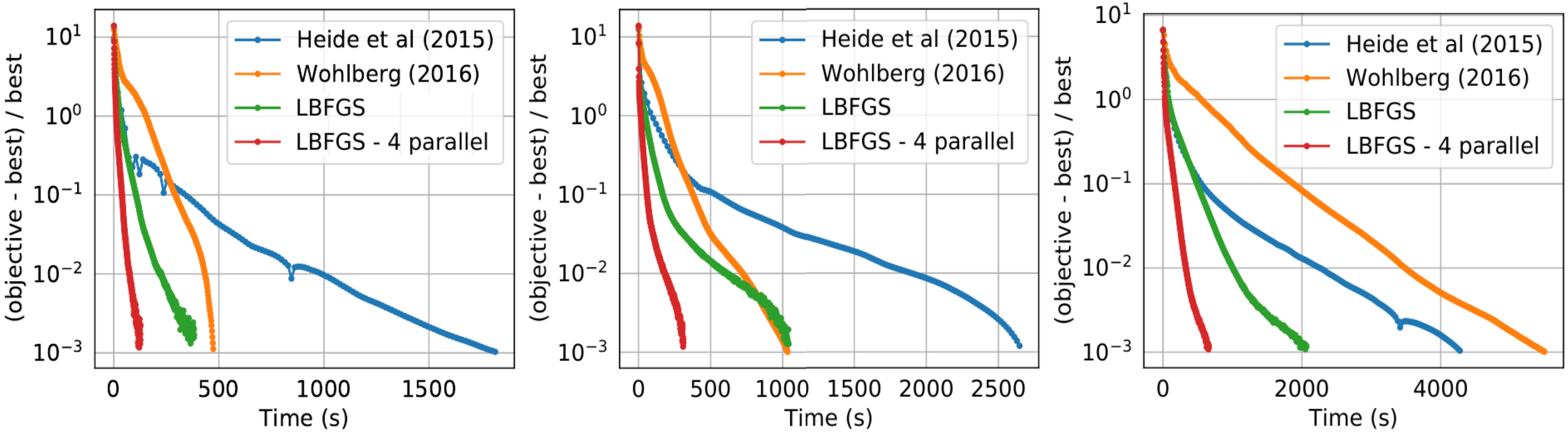
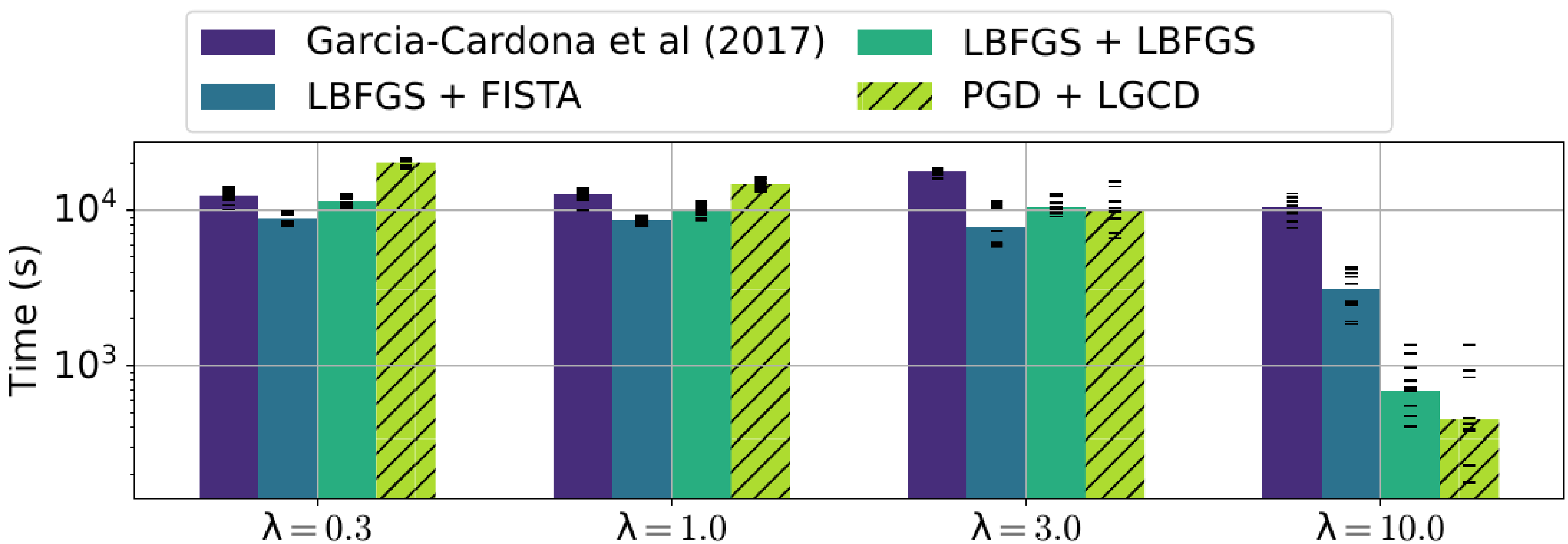
  - L-BFGS (Jas et al, 2017)
  - LGCD (Dupré la Tour et al, 2018)
- D-step
  - FFT (Grosse et al, 2007)
  - ADMM + FFT (Heide et al, 2015)
  - ADMM + FFT (Wohlberg, 2016)
  - L-BFGS (dual) (Jas et al, 2017)
  - PGD (Dupré la Tour et al, 2018)

Speed benchmarks



(a) $K = 2,\ L = 32.$ (b) $K = 2,\ L = 128.$ (c) $K = 10,\ L = 32.$



Second challenge: strong artifacts

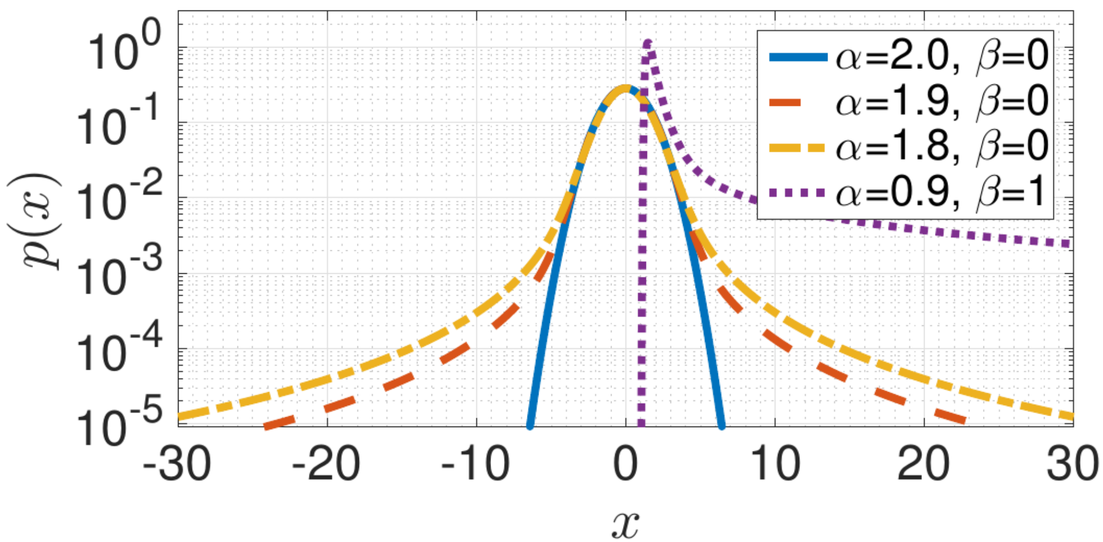
Gaussian CSC model

$$\hat{x}^n \triangleq \sum_{k=1}^{K} z_k^n * d_k$$

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad x^n[t] | z, d \sim \mathcal{N}(\hat{x}^n[t], 1),$$

Alpha-stable CSC model

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad x^n[t] | z, d \sim \mathcal{S}(\alpha, 0, 1/\sqrt{2}, \hat{x}^n[t])$$



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Alpha-stable CSC model

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad x^n[t] | z, d \sim \mathcal{S}(\alpha, 0, 1/\sqrt{2}, \hat{x}^n[t])$$

Conditional formulation (Samorodnitsky and Taqqu, 1994)

$$z_k^n[t] \sim \mathcal{E}(\lambda), \quad \phi^n[t] \sim \mathcal{S}\left(\frac{\alpha}{2}, 1, 2(\cos\frac{\pi\alpha}{4})^{2/\alpha}, 0\right)$$

$$x^n[t] | z, d, \phi \sim \mathcal{N}\left(\hat{x}^n[t], \frac{1}{2}\phi^n[t]\right)$$

Alpha CSC estimation

Monte Carlo Expectation-Maximization algorithm

- E-step: MCMC estimation (Chib and Greenberg, 1995)

$$w^n[t]^{(i)} \triangleq \mathbb{E}\left[1/\phi^n[t]\right]_{p(\phi|x,z^{(i)},d^{(i)})}$$

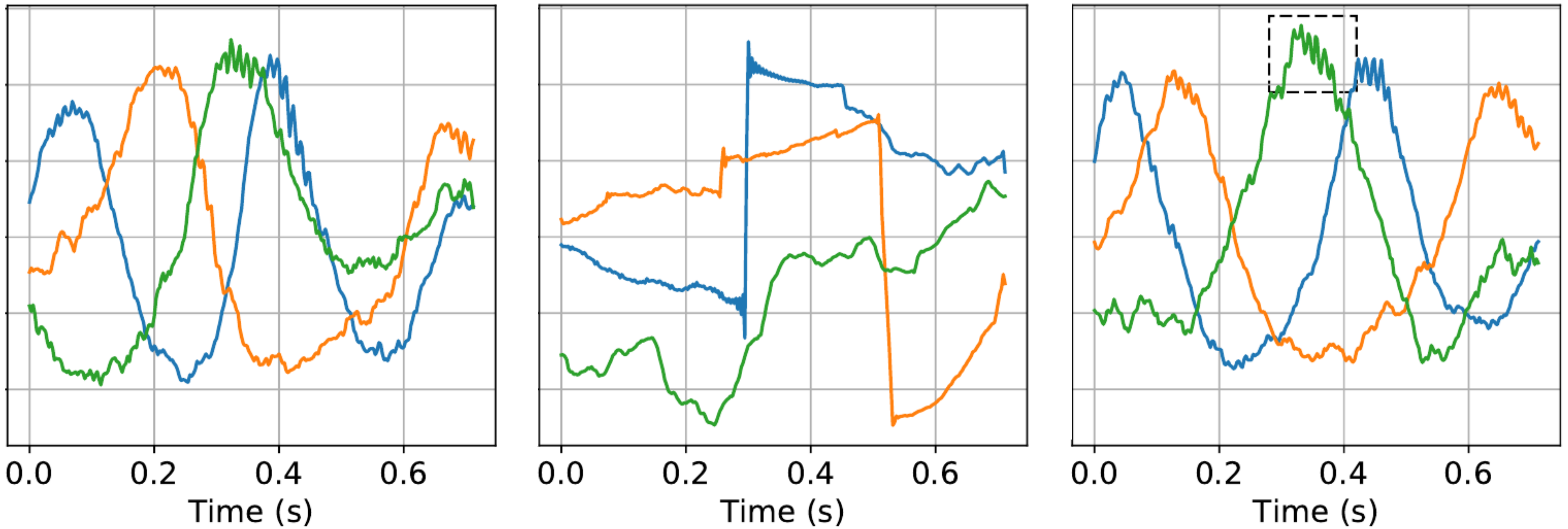
- M-step: weighted CSC

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| \sqrt{w^n} \odot (x^n - \sum_{k=1}^{K} z_k^n * d_k) \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1$$

$$\text{s.t.} \quad \|d_k\|_2^2 \leq 1, \text{ and } z_k^n \geq 0, \quad \forall k, n.$$

Learned atoms

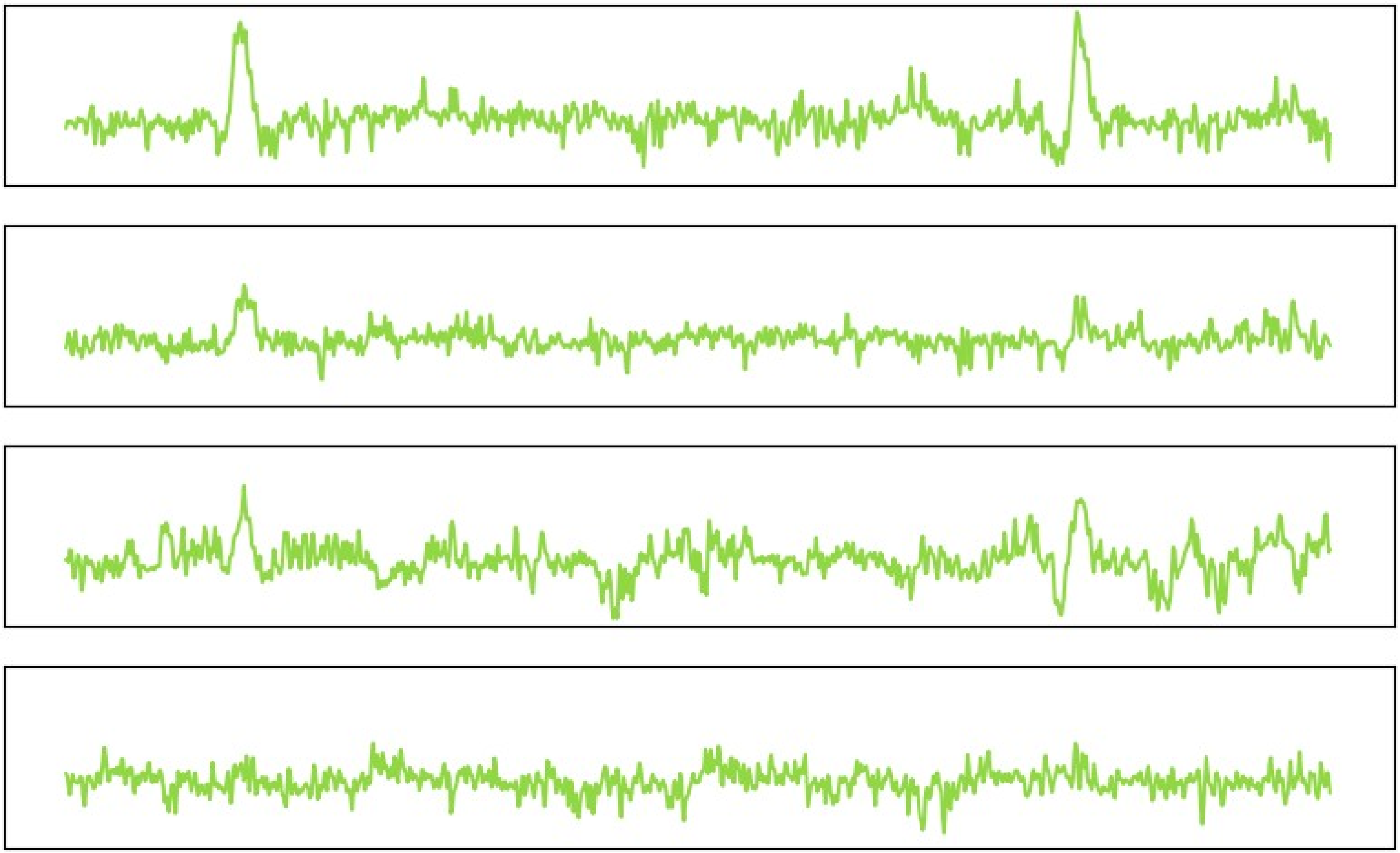
CSC (without artifacts) | CSC (with artifacts) | Alpha CSC (with artifacts)



Learning the morphology of brain signals using alpha-stable convolutional sparse coding
M. Jas, T. Dupré la Tour, U. Şimşekli, A. Gramfort, *NeurIPS 2017*

Third challenge: multivariate models

MEG



Third challenge: multivariate models

$$\min_{D,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * D_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$

$$\text{s.t.} \quad \|D_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

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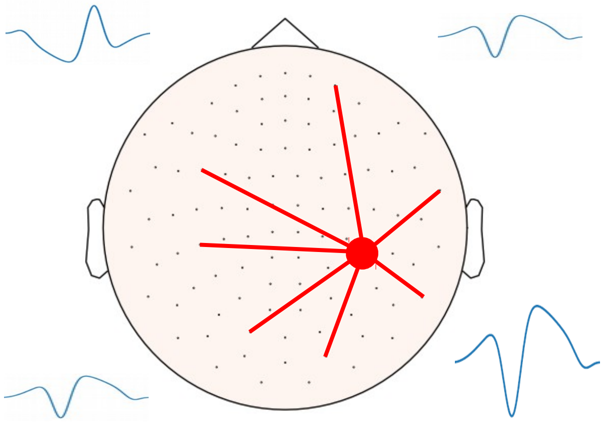
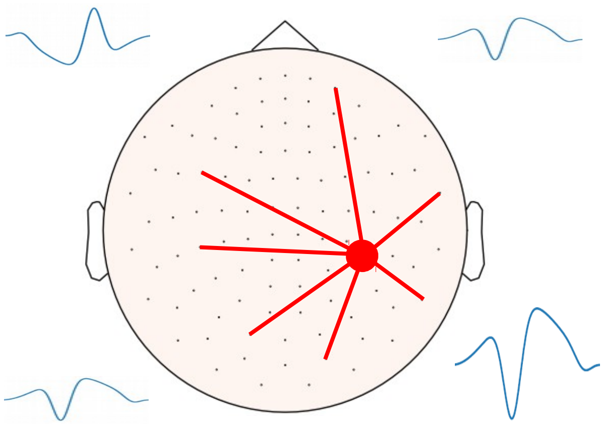


Third challenge: multivariate models

$$\min_{u,v,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * (u_k v_k^\top) \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1 ,$$
$$\text{s.t.} \quad \|u_k\|_2^2 \leq 1 \ , \ \|v_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

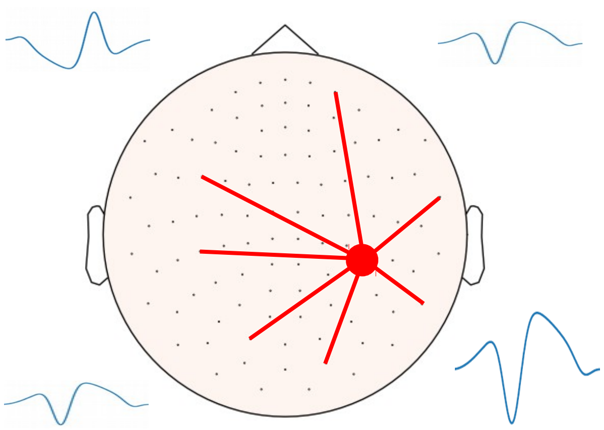


Third challenge: multivariate models

$$\min_{u,v,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * (u_k v_k^\top) \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1 ,$$
$$\text{s.t.} \quad \|u_k\|_2^2 \leq 1 \ , \ \|v_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

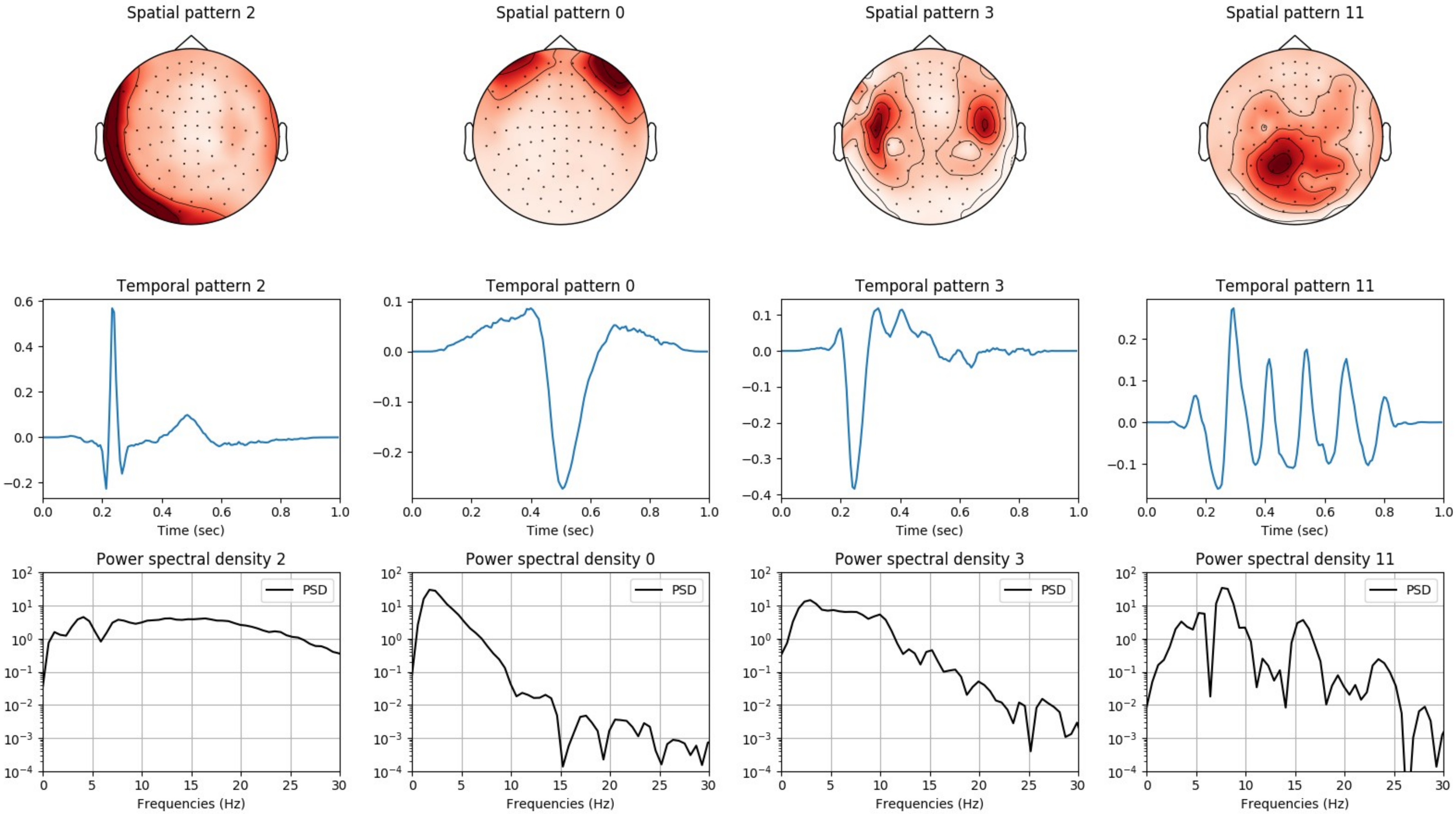
Rank-1 constraint

- Consistent with Physics of EM waves
- Scales in (L+P) instead of (LP)

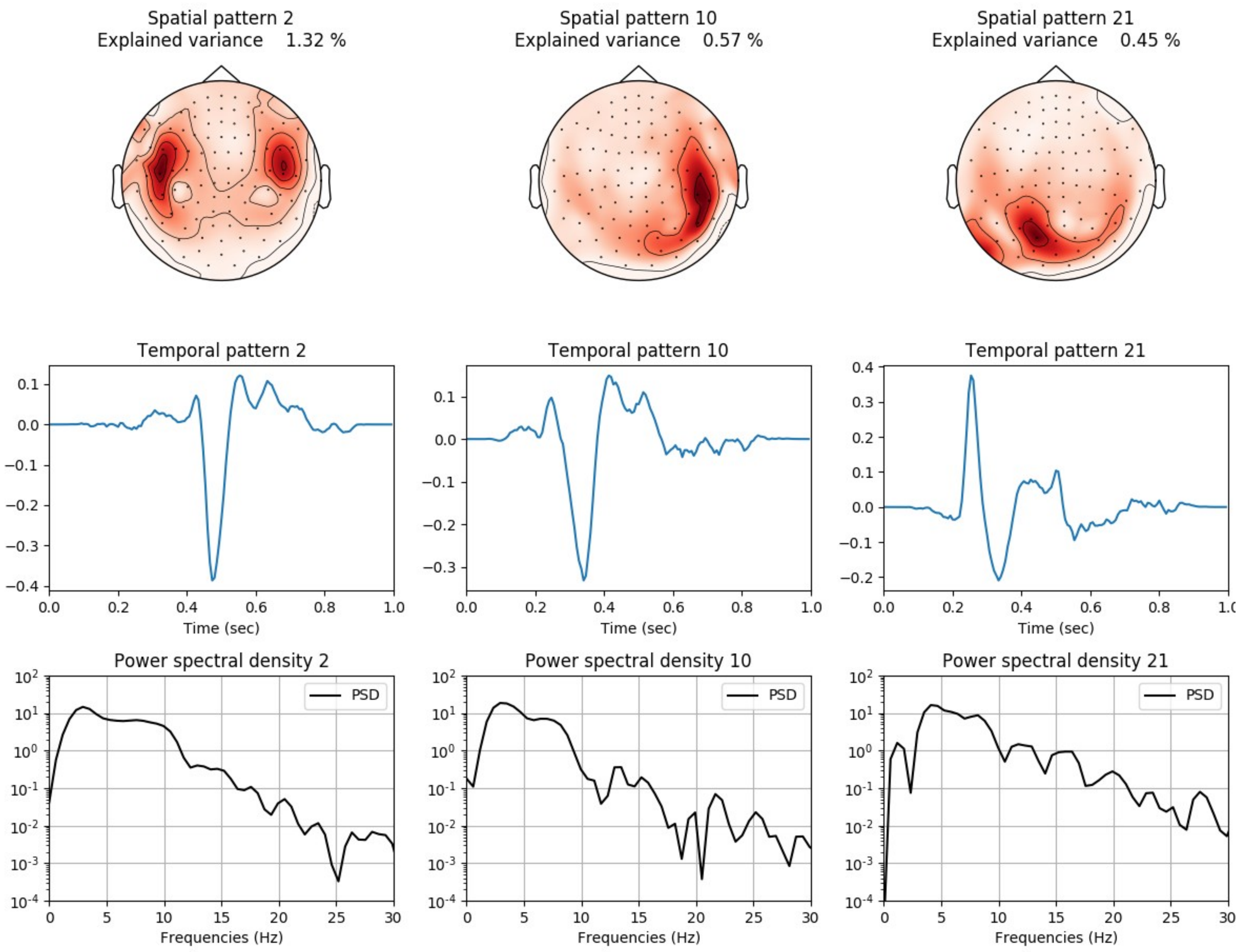


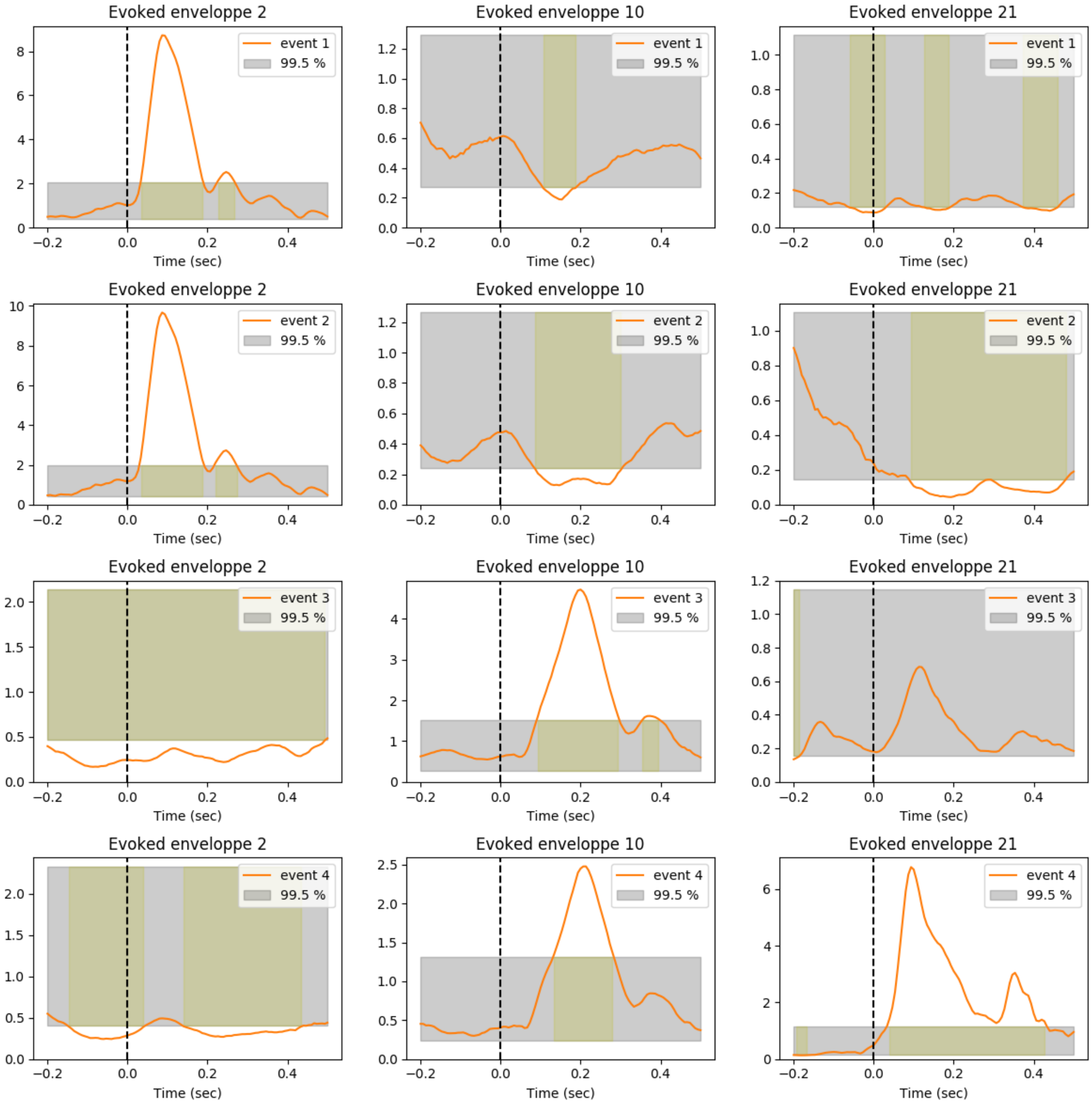
Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals
T. Dupré la Tour*, T. Moreau*, M. Jas, A. Gramfort, *NeurIPS 2018*

Multivariate atoms



Event-related atoms




Evoked enveloppe 2
event 1
99.5 %
Time (sec)
Evoked enveloppe 10
event 1
99.5 %
Time (sec)
Evoked enveloppe 21
event 1
99.5 %
Time (sec)
Evoked enveloppe 2
event 2
99.5 %
Time (sec)
Evoked enveloppe 10
event 2
99.5 %
Time (sec)
Evoked enveloppe 21
event 2
99.5 %
Time (sec)
Evoked enveloppe 2
event 3
99.5 %
Time (sec)
Evoked enveloppe 10
event 3
99.5 %
Time (sec)
Evoked enveloppe 21
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99.5 %
Time (sec)
Evoked enveloppe 2
event 4
99.5 %
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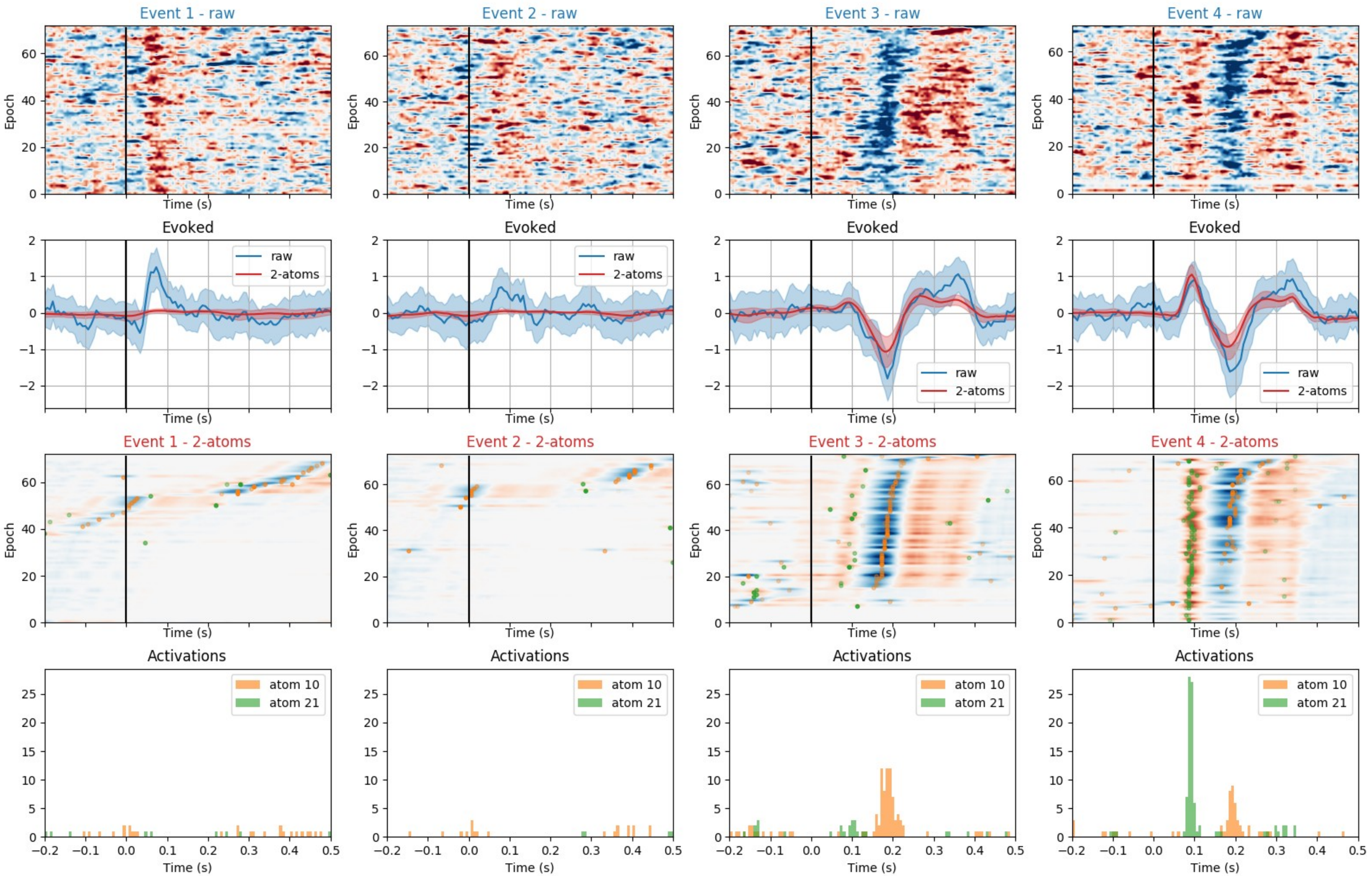
Evoked enveloppe 10
event 4
99.5 %
Time (sec)
Evoked enveloppe 21
event 4
99.5 %
Time (sec)

Event-related activations



Temporal waveform analysis

with convolutional sparse coding models

- CSC well-posed optimization problem
- Alpha CSC model for robustness to strong artifacts
- Multivariate CSC model, rank-1 constraint

- Open-source implementation
  - with unit tests, documentation, examples

`https://alphacsc.github.io`