Non-linear auto-regressive models for cross-frequency coupling in neural time series

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https://github.com/pactools/pactools

0. Abstract

- In neuroscience, *phase-amplitude coupling* (PAC) refers to the interaction between the phase of a slow neural oscillation and the amplitude of high frequencies, within the same signal or across two signals.
- To model PAC, we use *new* parametric *driven auto-regressive* (DAR) models. These generative statistical models provide a non-linear estimation of the amplitude modulation over the entire spectrum, avoiding the pitfalls related to incorrect filtering or the use of the Hilbert transform on wide-band signals.
- As the model is probabilistic, it also provides a score of the model "goodness of fit" via the *likelihood*, enabling easy and legitimate parameter comparison; this data-driven feature is unique to our model-based approach.
- We show how the likelihood can be used to find optimal filtering parameters, suggesting new properties on the spectrum of the driving signal, and also to estimate the optimal delay between the coupled signals, enabling a directionality estimation of the coupling.

1. Driven auto-regressive (DAR) models

- AR model $$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t)$$
- DAR model $$a_i(t) = \sum_{j=0}^{m} a_{ij} x(t)^j, \quad \log(\sigma(t)) = \sum_{j=0}^{m} b_j x(t)^j$$

Maximum Likelihood Estimate :

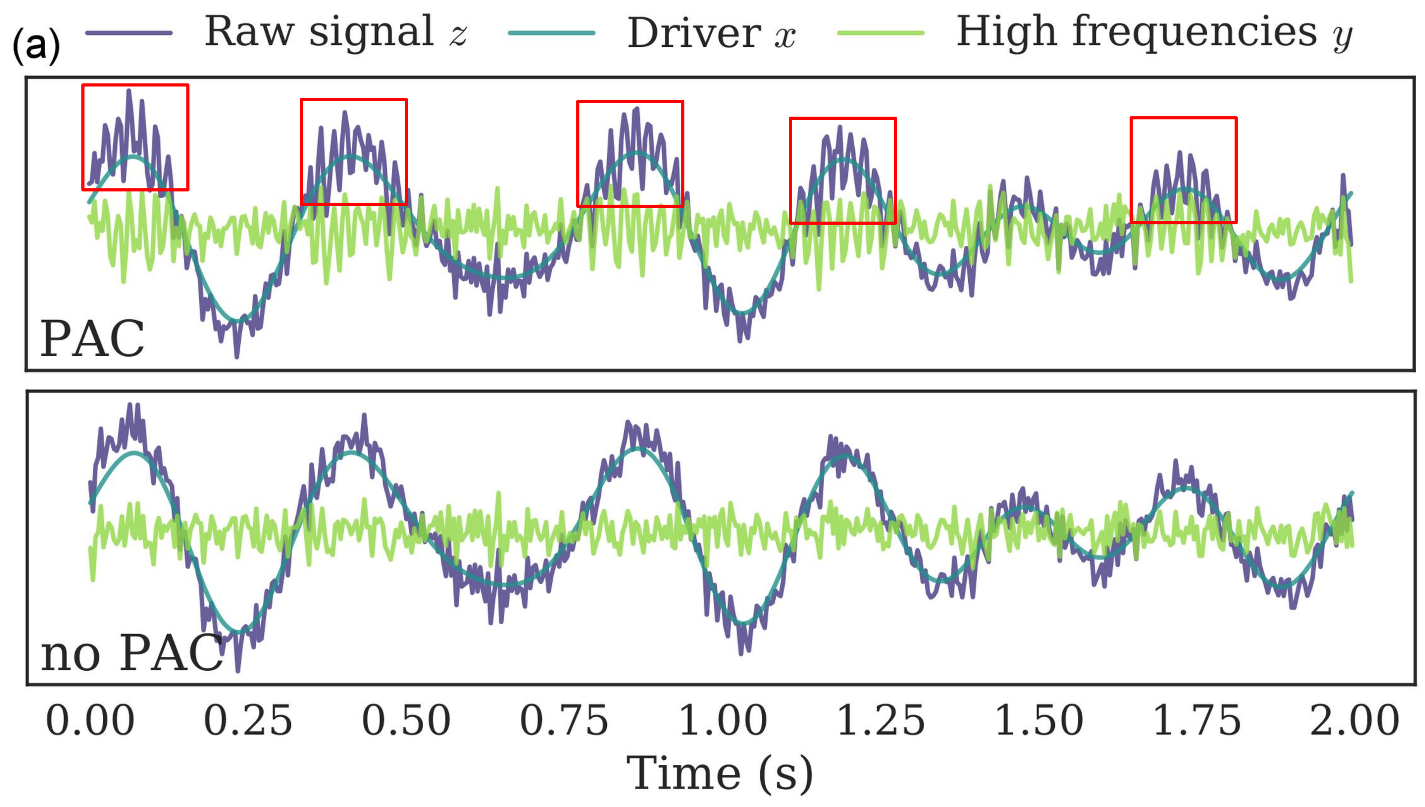
- Linear system for the AR coefficients a_{ij}
- Newton-Raphson for the gain coefficients b_j

- Likelihood $$L = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

2. Phase-amplitude coupling (PAC)

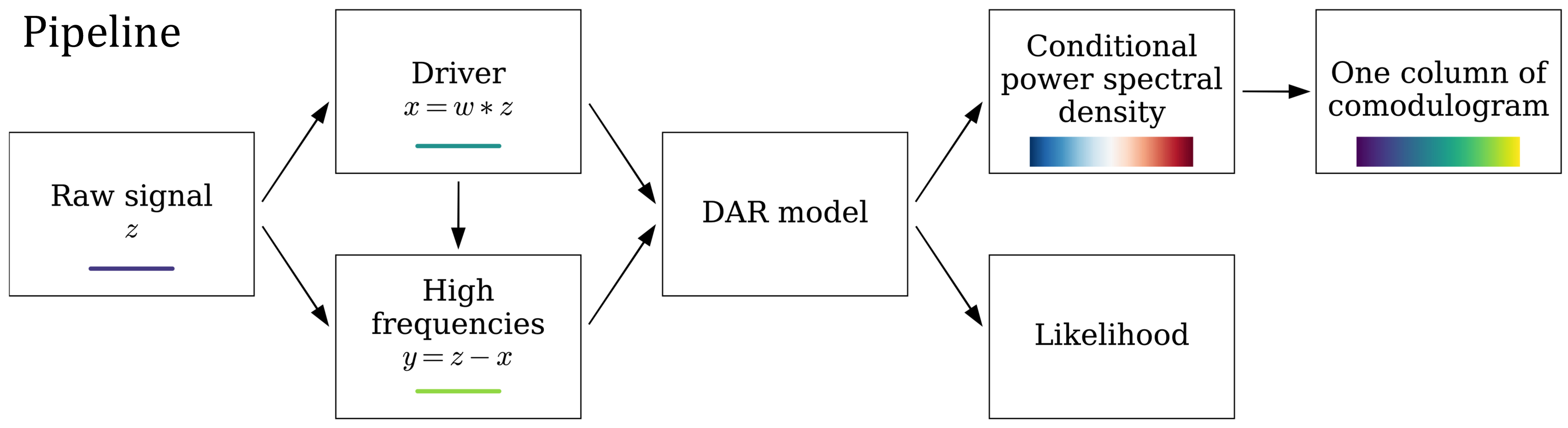
It's a coupling between :

- The phase of a slow oscillation
- The amplitude of high frequencies

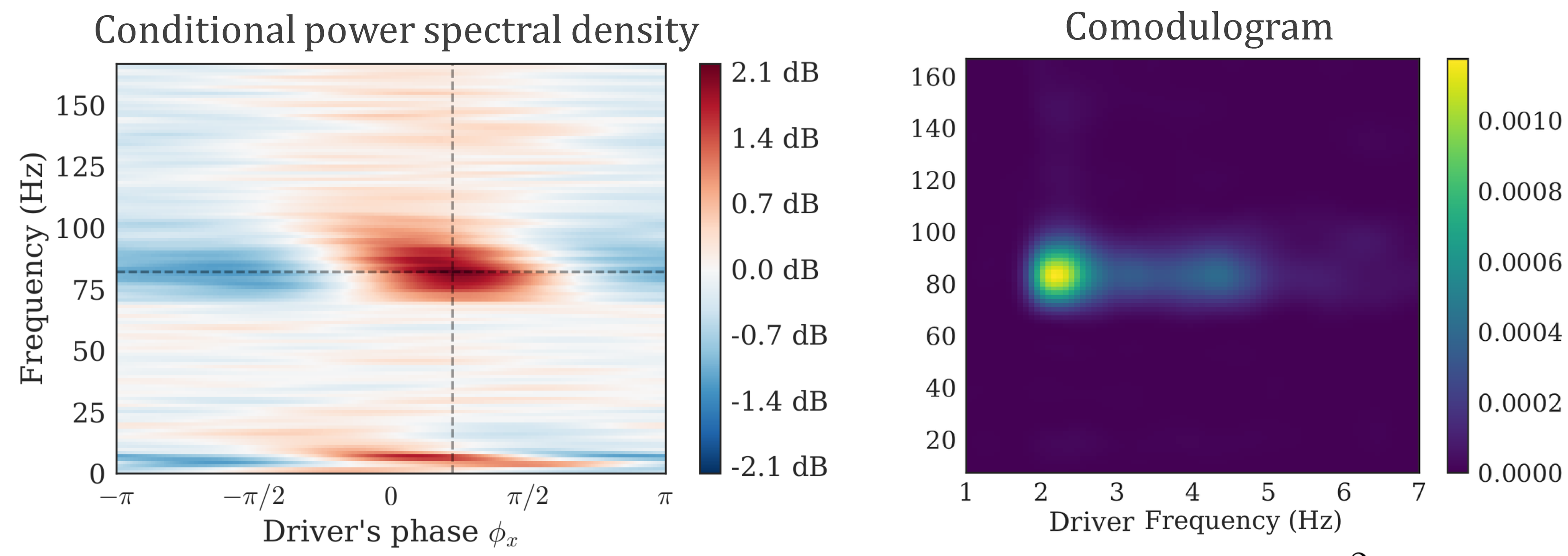


3. Power spectral density (PSD) and comodulogram

- Pipeline



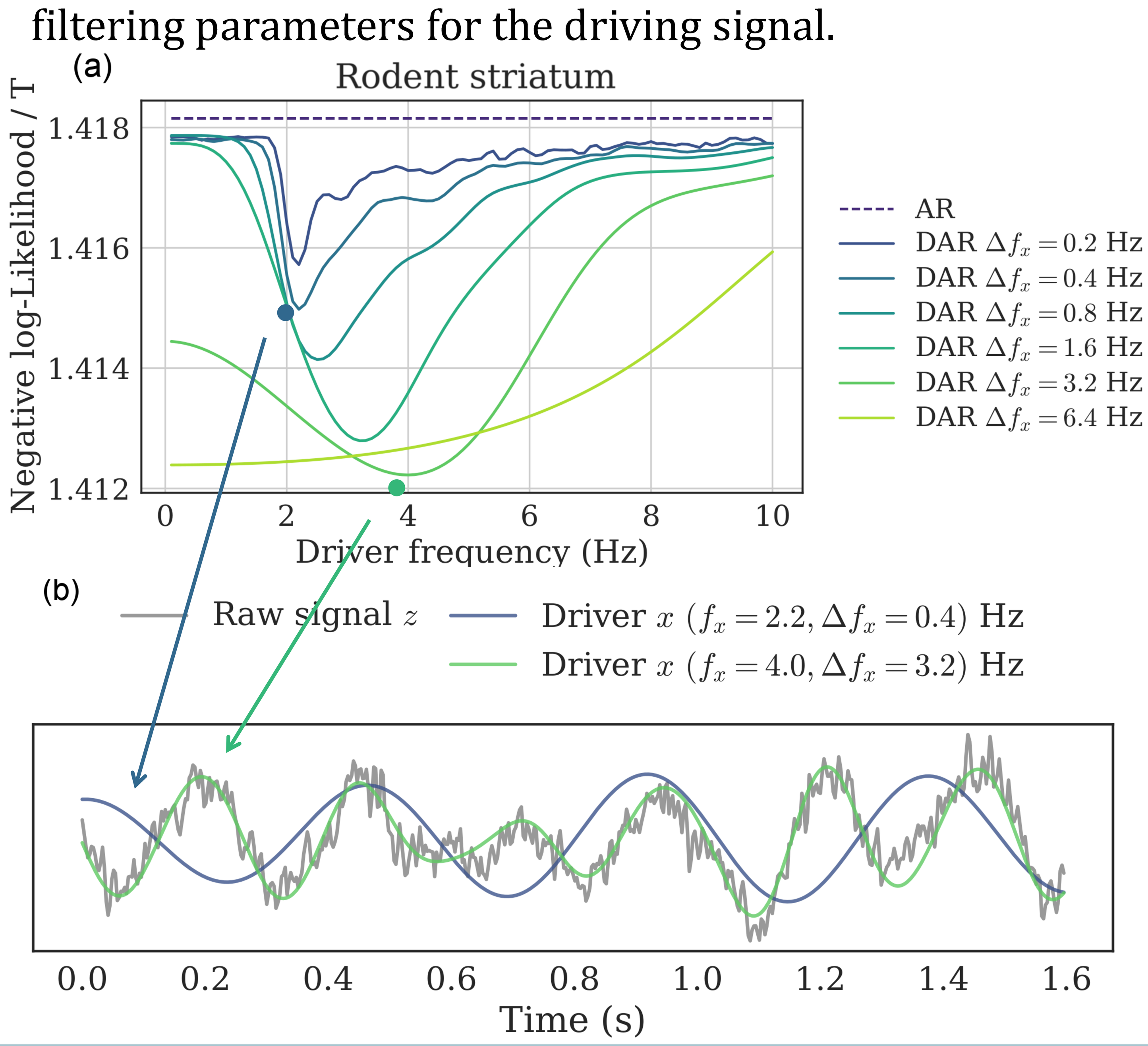
- On rodent striatum (LFP)



- Power spectral density $$S_y(x_0)(f) = \left| \sum_{i=0}^{p} \frac{a_i(x_0)}{\sigma(x_0)} e^{-j2\pi f i} \right|^{-2}$$

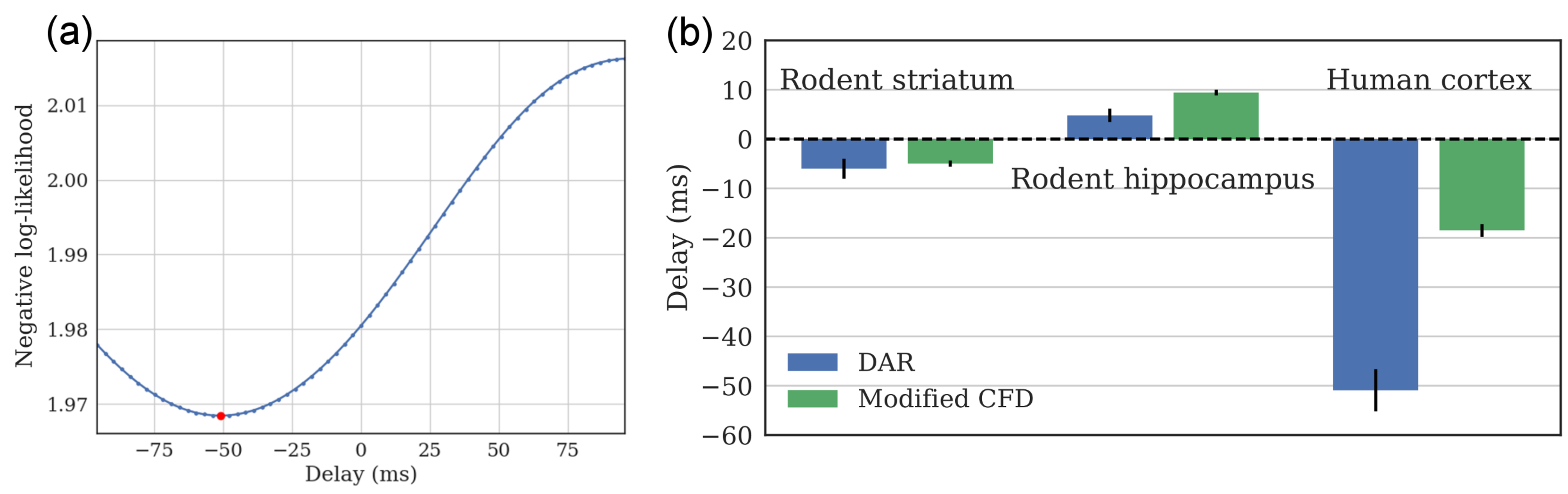
4. Filtering parameters selection

- Optimizing the model likelihood, we estimate the optimal filtering parameters for the driving signal.



5. Delay estimation

- Optimizing the model likelihood, we estimate the optimal delay between the signal and the driver, to get the directionality of the coupling.



References

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