Learning the morphology of Brain Signals Using Alpha-Stable Convolutional Sparse Coding

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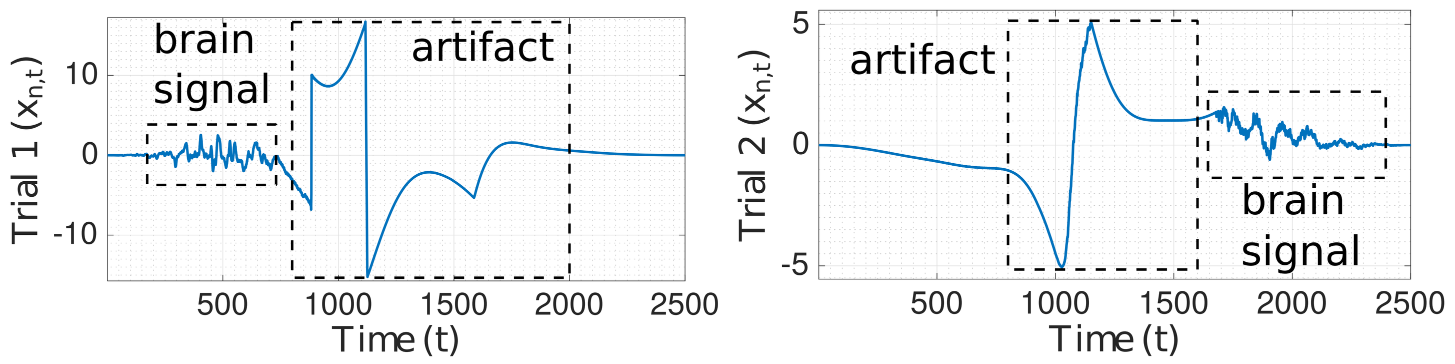
Motivation

- Learn prototypical signals
- Shape of signal is critical in clinical settings (Cole, 2017)
- Convolutional Sparse Coding (CSC) mathematically motivated. But ... neural signals are noisy



Problem formulation

Learn atoms $d \equiv \{d^k\}_k$, activations $z \equiv \{z_n^k\}_{n,k}$ from signal x_n

$$(d^\star, z^\star) = \underset{d,z}{\arg\min} \sum_n \left(\|x_n - \sum_k d^k * z_n^k\|_2^2 + \lambda \sum_k z_n^k \right) \qquad (1)$$

s.t. $z_n^k \geq 0$ and $\|d^k\|_2^2 \leq 1$

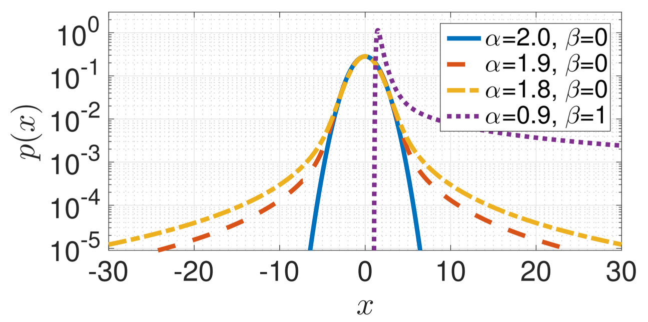
Data likelihood:

'Vanilla' CSC: $x_{n,t}|z,d \sim \mathcal{N}(\sum_k d^k * z_n^k, 1)$

αCSC: $x_{n,t}|z,d \sim \mathcal{S}(\alpha, \beta = 0, \sigma = 1/\sqrt{2}, \sum_k d^k * z_n^k)$

Alpha-stable distribution

$\mathcal{S}(\alpha, \beta, \sigma, \mu)$ is characterized by 4 parameters



- Characteristic exponent $\alpha \in (0, 2]$
- Skewness $\beta \in [-1, 1]$
- Location $\mu \in (-\infty, +\infty)$
- Scale $\sigma \in (0, \infty)$

Special case: $\mathcal{S}(\alpha = 2, \beta = 0, \sigma, \mu) = \mathcal{N}(\mu, 2\sigma^2)$

Conditionally Gaussian: ($\beta = 0$)

$$x_{n,t}|z,d \sim \mathcal{N}\left(\sum_{k=1}^K d^k * z_n^k, \frac{1}{2}\phi_{n,t}\right)$$

Weighted least squares!

and $\phi_{n,t}$ comes from a positive stable distribution

But $\phi_{n,t}$ is unknown! $\Longrightarrow$ use EM algorithm

Monte Carlo EM for αCSC

E-Step: $\mathcal{B}^{(i)}(d,z) = \mathbb{E}\left[\log p(x, \phi, z|d)\right]_{p(\phi|x,z^{(i)},d^{(i)})}$, where

$$\mathcal{B}^{(i)}(d,z) =^{+} -\sum_{n=1}^N \left(\|\sqrt{w_n^{(i)}} \odot (x_n - \sum_{k=1}^K d^k * z_n^k)\|_2^2 + \lambda \sum_{k=1}^K z_n^k \right)$$

where $w_{n,t}^{(i)} \triangleq \mathbb{E}\left[1/\phi_{n,t}\right]_{p(\phi|x,z^{(i)},d^{(i)})}$

MCMC approximation:

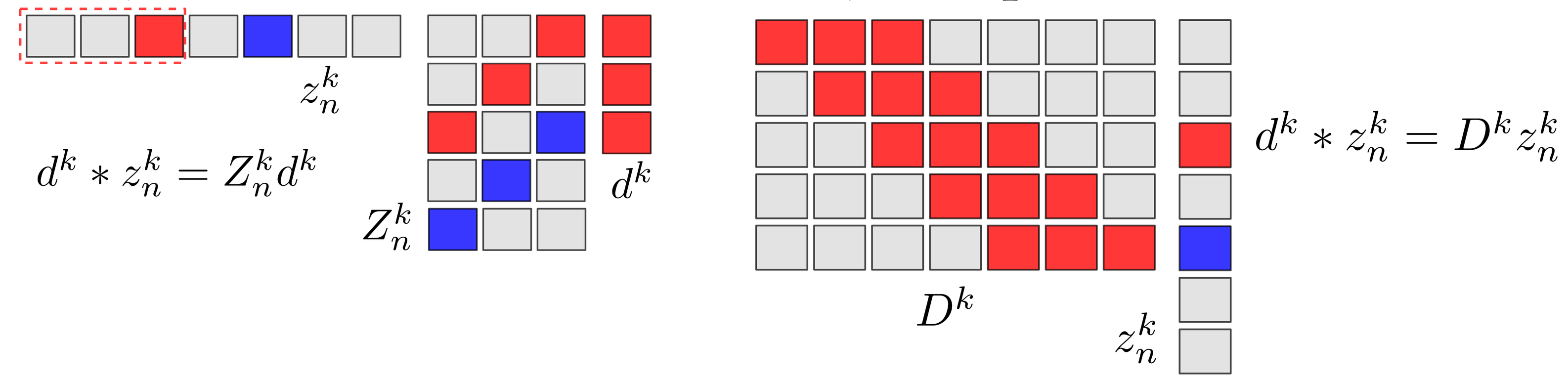
$$w_{n,t}^{(i)} \approx (1/J) \sum_{j=1}^J 1/\phi_{n,t}^{(i,j)}$$

since ϕ is not known analytically but can be sampled from.

M-Step: $(d^{(i+1)}, z^{(i+1)}) = \arg\max_{d,z} \mathcal{B}^{(i)}(d,z)$

Alternate minimization:

d step: Strided matrix *z step:* Toeplitz matrix



Algorithm

Algorithm 1: α-stable CSC

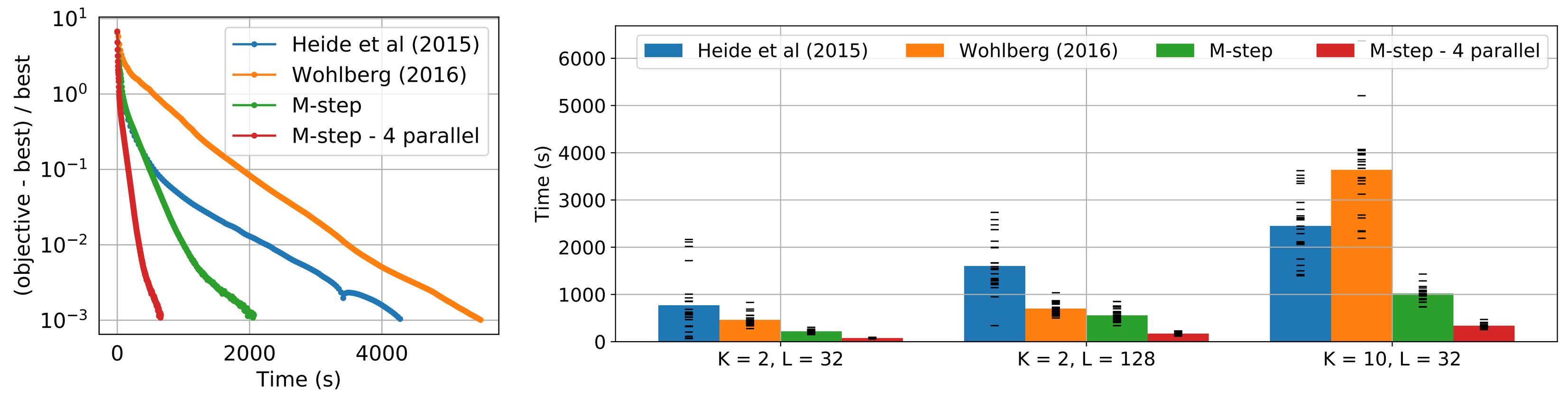
Require: Regularization: $\lambda \in \mathbb{R}_+$

for $i = 1$ to I **do**
  /* E-step: */
  for $j = 1$ to J **do**
    Draw $\phi_{n,t}^{(i,j)}$ via MCMC
  end for
  $w_{n,t}^{(i)} \approx (1/J) \sum_{j=1}^J 1/\phi_{n,t}^{(i,j)}$
  /* M-step: */
  for $m = 1$ to M **do**
    $z^{(i)}$ = L-BFGS-B with Toeplitz matrix
    $d^{(i)}$ = Block CD + L-BFGS-B with strided matrix
  end for
end for
return $w^{(I)}, d^{(I)}, z^{(I)}$

Results

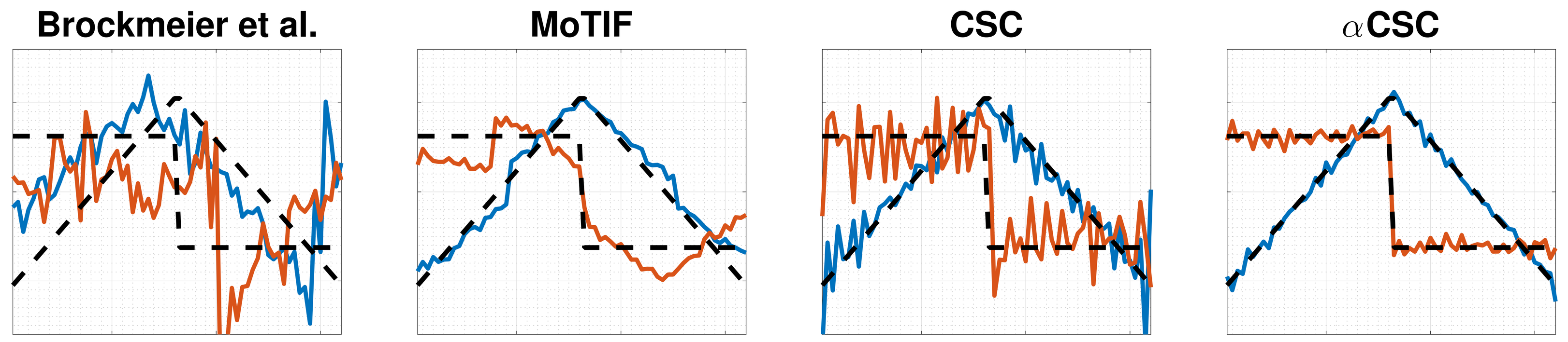
Speed benchmarks ($\alpha = 2$)

- Quasi-Newton solvers beats state-of-art ADMM solvers in CSC from computer vision



Simulations

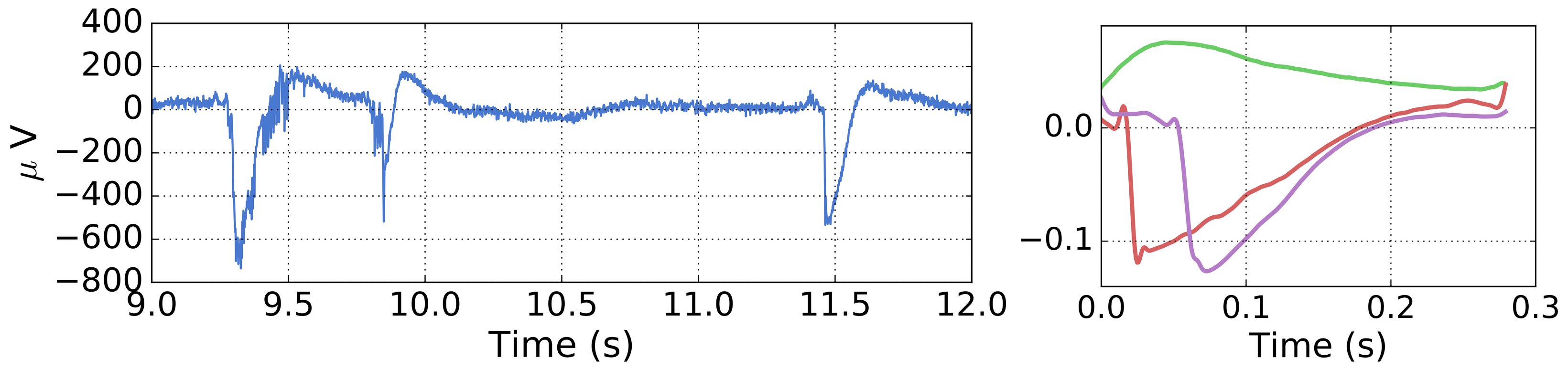
- 10% trials corrupted
- αCSC is more robust to artifacts on simulated data



Real data

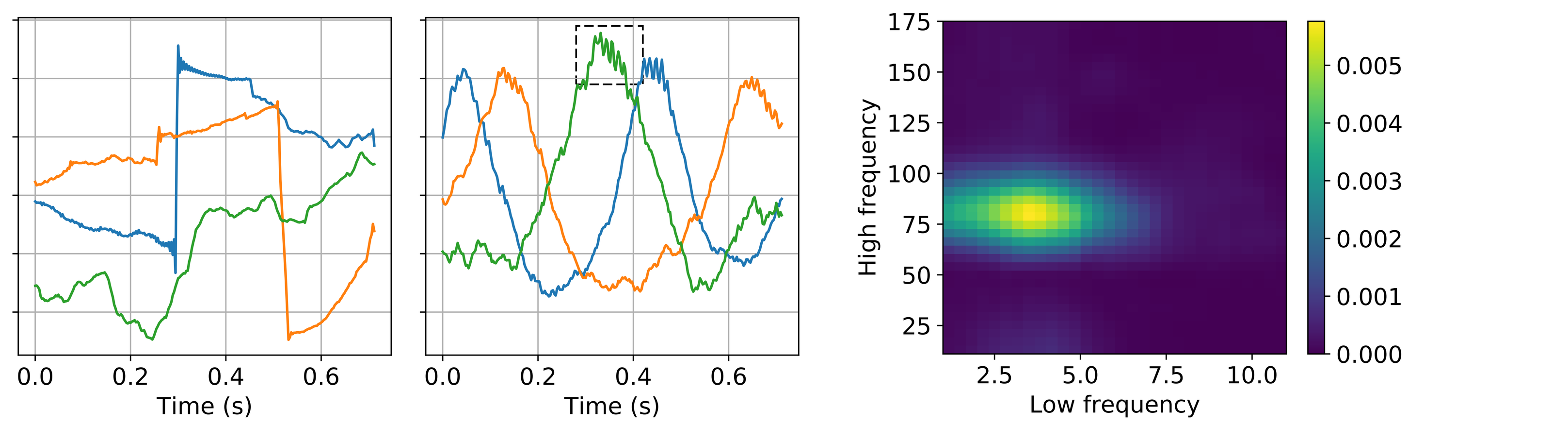
Learning epiliptiform spikes from data ($\alpha = 0$)

- Atoms recovered resemble spikes in the time series



Discovering nested oscillations ($\alpha = 1.2$)

- Data from rodent striatum
- Estimated atoms on 'Vanilla' CSC is affected by artifacts
- Atoms containing 80 Hz oscillations nested within 2 Hz oscillations found with αCSC



Acknowledgement

French National Research Agency grants ANR-14-NEUC-0002-01 and ANR-16-CE23-0014, and the ERC Starting Grant SLAB ERC-YStG-676943

References

- Cole, S. R. and Voytek, B. (2017). Brain Oscillations and the Importance of Waveform Shape. Trends Cogn. Sci.
- Source code available at `http://alphacsc.github.io`