0. Abstract

- In this paper, we introduce new parametricand generative driven auto-regressive (DAR) models. DAR models provide a non-linear and non-stationary spectral estimation of a signal, conditionally to another exogenous signal.
- We detail how inference can be done efficiently while guaranteeing model stability. We show how model comparison and hyper-parameter selection can be done using likelihood estimates. We also point out the limits of DAR models when the exogenous signal contains too high frequencies.
- Finally, we illustrate how DAR models can be applied on neuro-physiologic signals to characterize phase-amplitude coupling.

2. Model selection

- Model likelihood
  \[ L = \prod_{t \in P} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp \left( -\frac{(z(t))^2}{2\sigma(t)^2} \right) \]

- Bayesian information criterion (BIC)
  \[ BIC = -2\log(L) + d\log(T) \]
  where \( d = (p + 1)(m + 1) \)

- Simulations: we create DAR model, synthetize a driver and a signal, and try to recover the model order with BIC selection.

3. Application to neuroscience

In neuroscience, phase-amplitude coupling refers to the interaction between:
- The phase of a slow neural oscillation \( x \)
- The amplitude of high frequencies \( y \)

**Example of a signal from human electro-physiology**

We band-pass filter the driver \( x \) from the signal, and apply DAR models on the high frequencies \( y \), to estimate the PAC.

**The PSD varies as a function of the driver**

4. Comparison with LSTAR

- Logistic smooth-transition AR
  \[ a_i(t) = \sum_{j=0}^{m} a_{ij} F_j(x(t)) \]
  \[ F_j(x(t)) = \left( 1 + e^{-c_j(x(t) - c_{ij})} \right)^{-1} \]
- For fair comparison we added
  \[ \log(\sigma(t)) = \sum_{j=0}^{m} b_j F_j(x(t)) \]

**BIC comparison on a signal from human electro-physiology**

**References**