Non-linear auto-regressive models for cross-frequency coupling in neural time series

Tom Dupré la Tour

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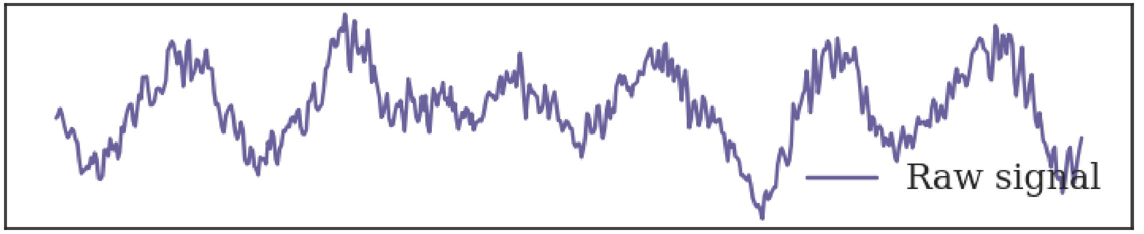
Joint work with: Lucille Tallot, Laetitia Grabot, Valérie Doyère, Virginie van Wassenhove, Yves Grenier, Alexandre Gramfort

Cross-frequency coupling in neural time series

Neural time series: Local field potential in rodent striatum



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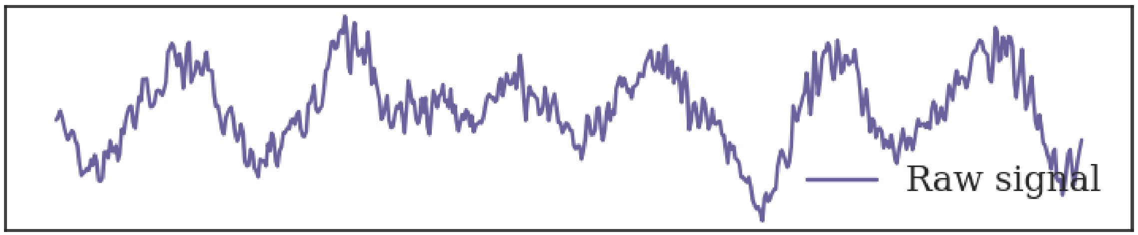

Cross-frequency coupling (CFC)

- Phase-amplitude coupling (PAC)
- Phase-phase coupling
- Phase-frequency coupling
- Amplitude-amplitude coupling

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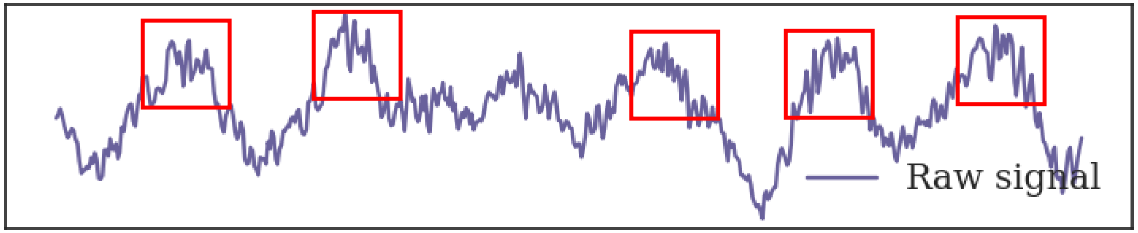

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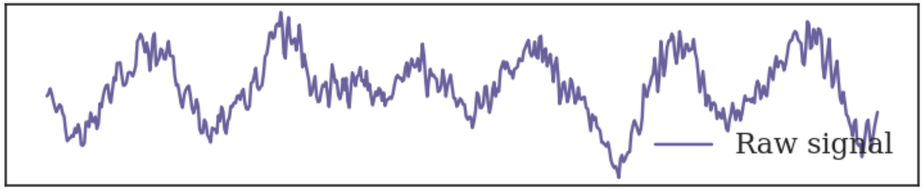
Phase Amplitude Coupling (PAC) metrics



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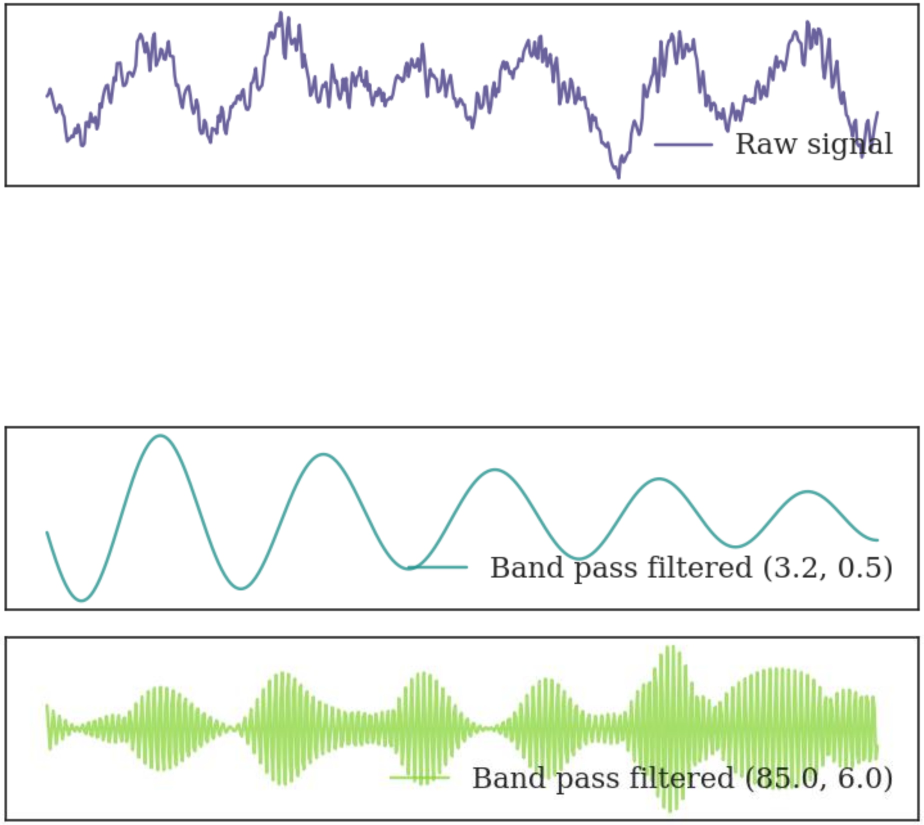

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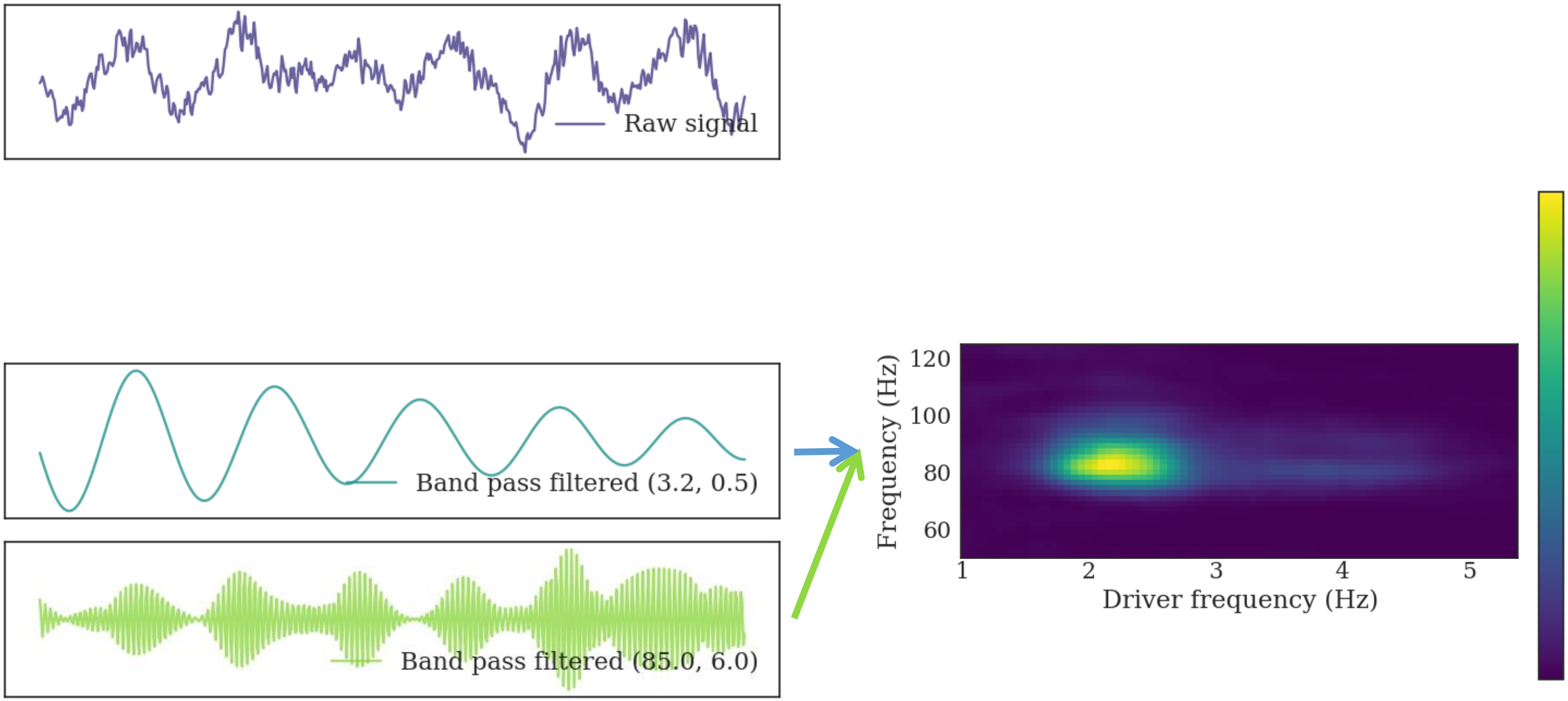

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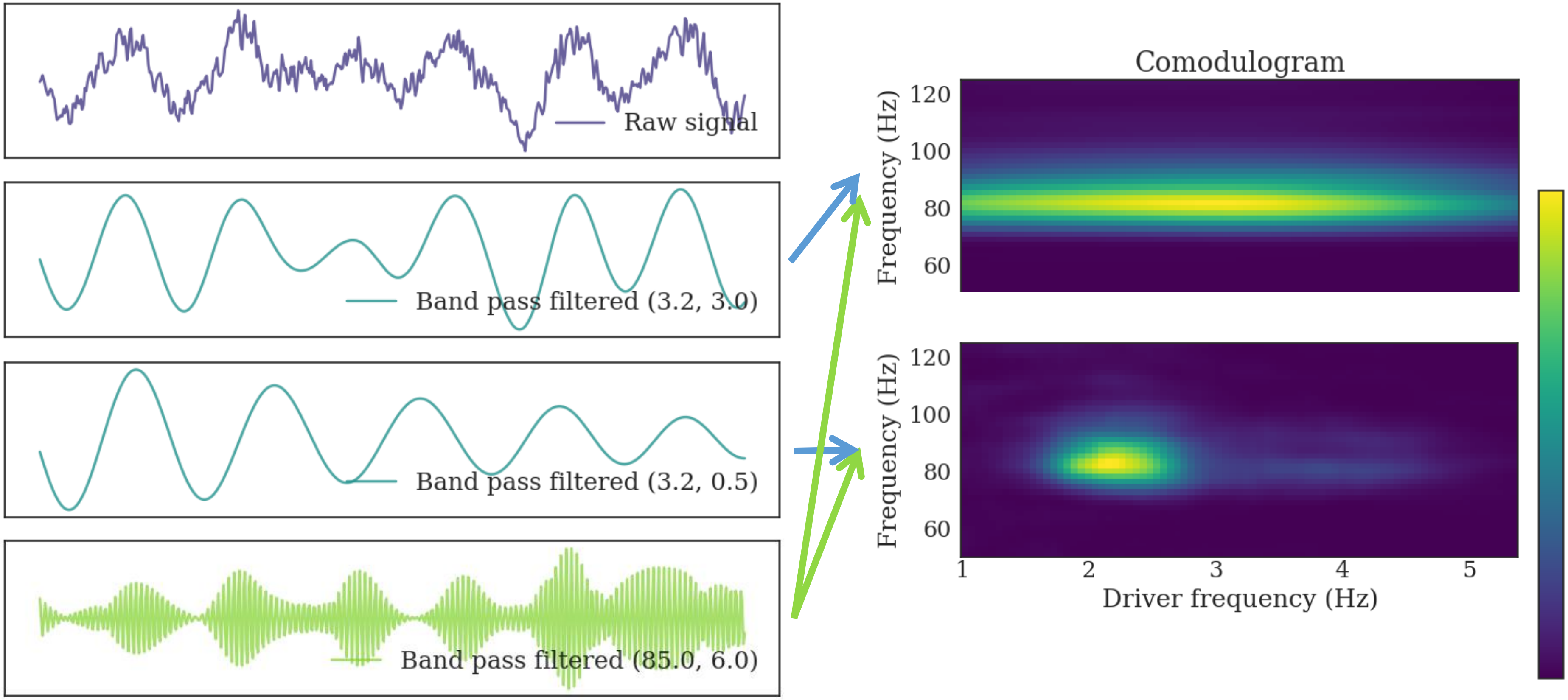

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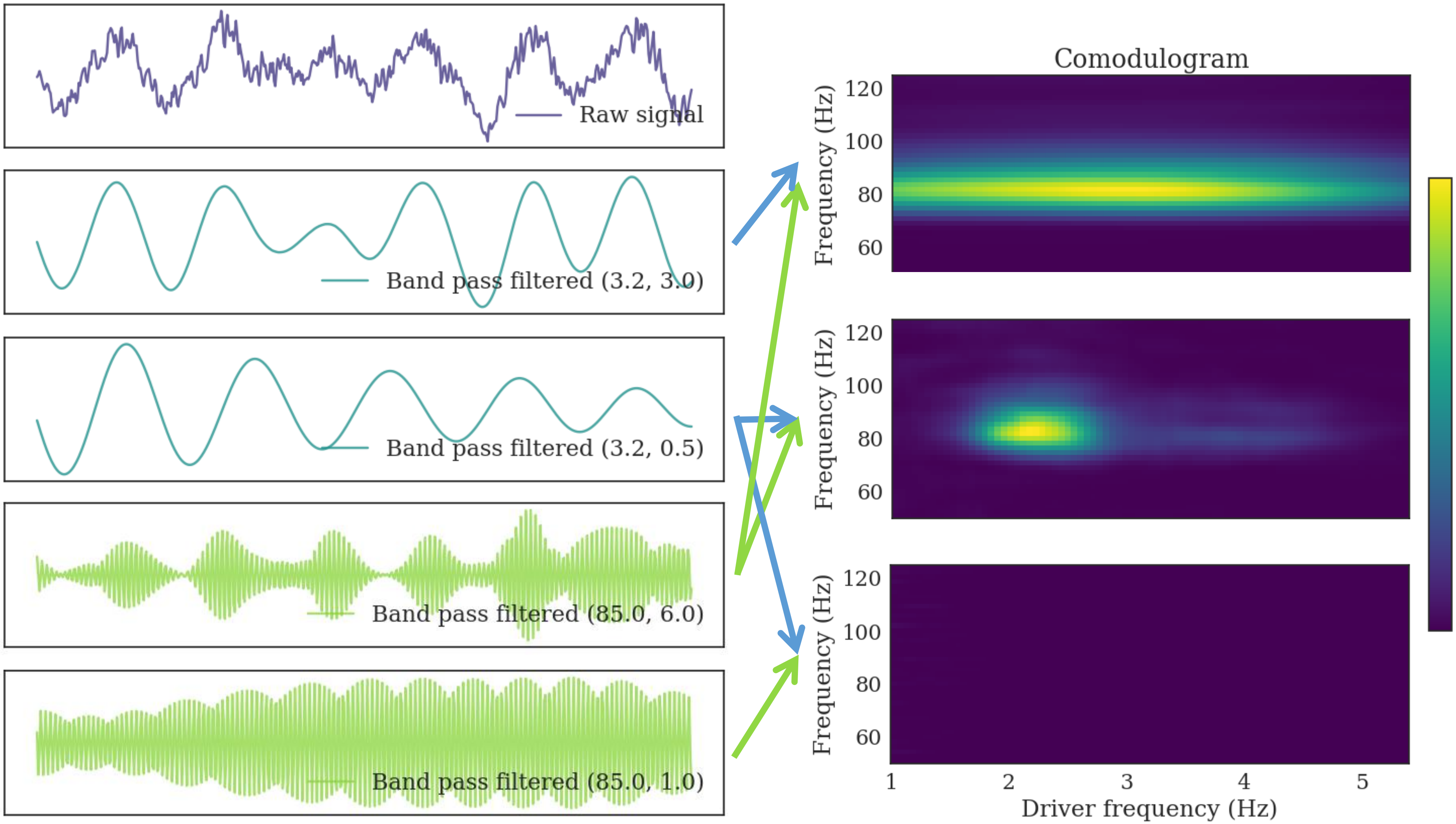

Auto-Regressive (AR) model

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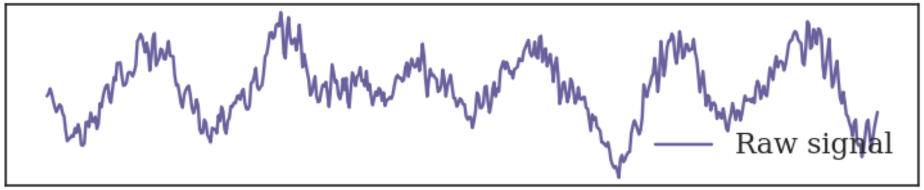

Driven Auto-Regressive (DAR) model

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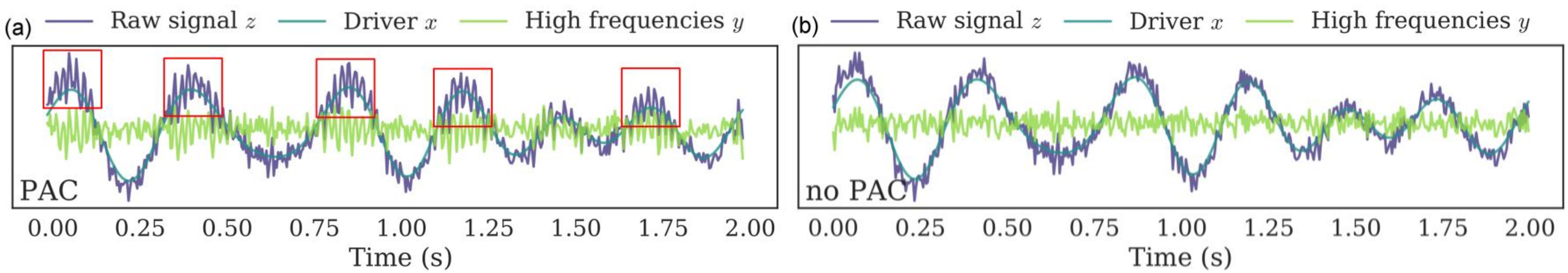
- Driven AR (DAR) model

$$a_i(t) = \sum_{j=0}^{m} a_{ij} x(t)^j \qquad \log(\sigma(t)) = \sum_{j=0}^{m} b_j x(t)^j$$

Power Spectral Density

$$\mathrm{PSD}_y(x_0)(f) = \sigma(x_0)^2 \left| \sum_{i=0}^{p} a_i(x_0) e^{-j2\pi f i} \right|^{-2}$$



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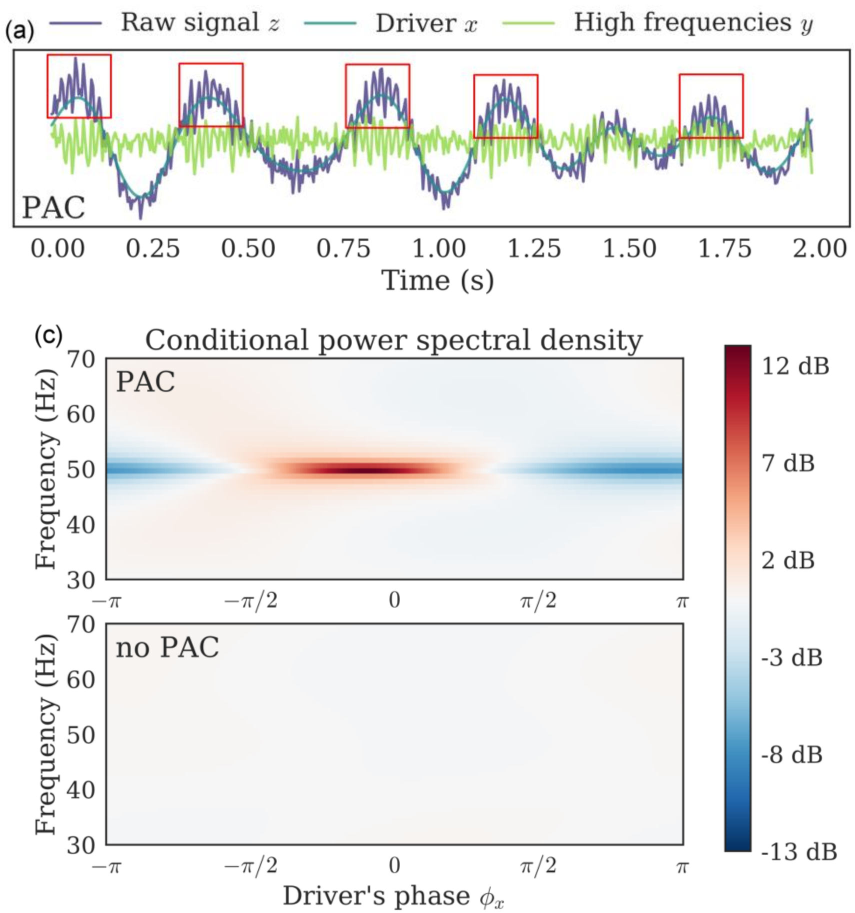

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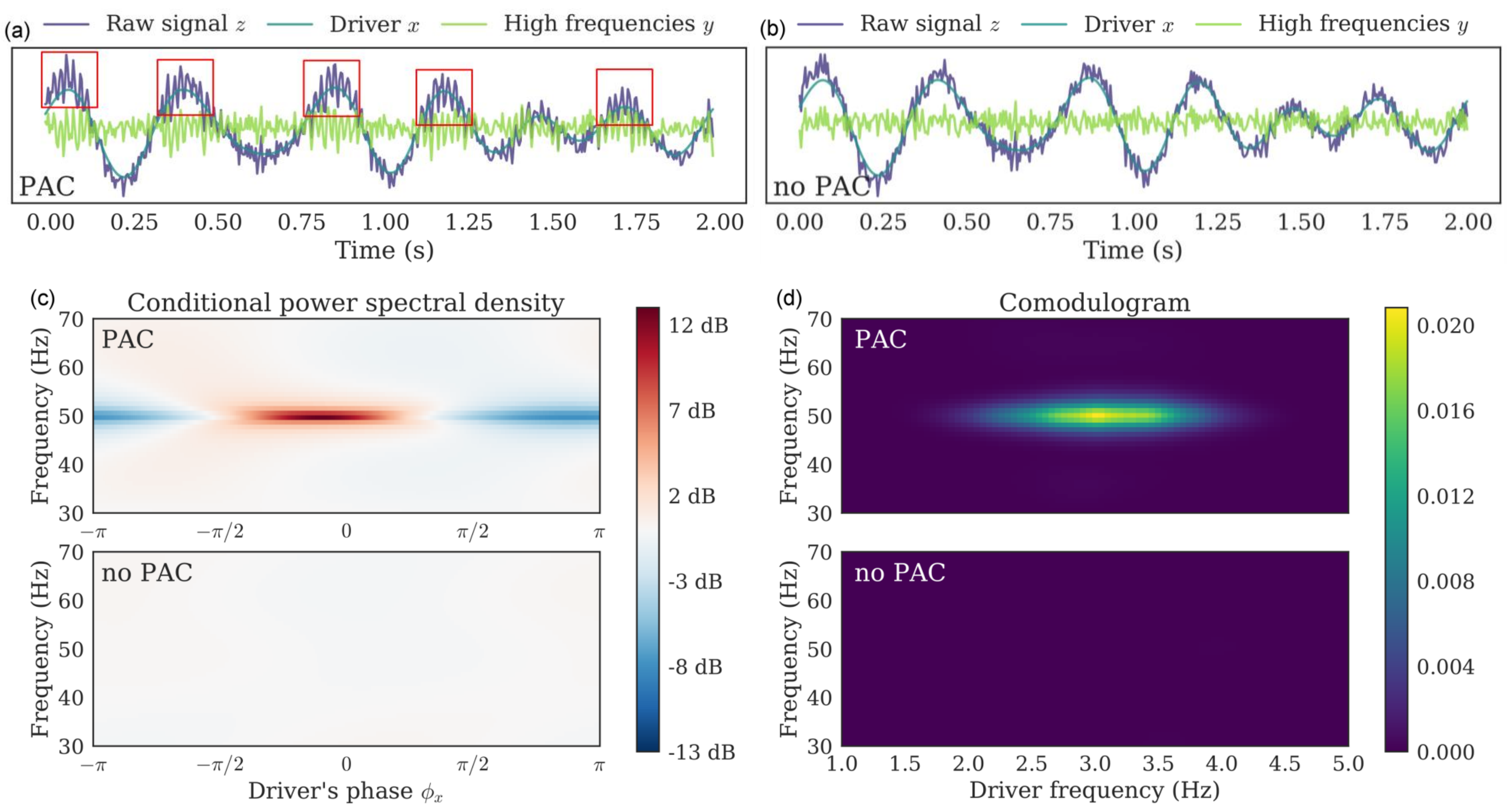
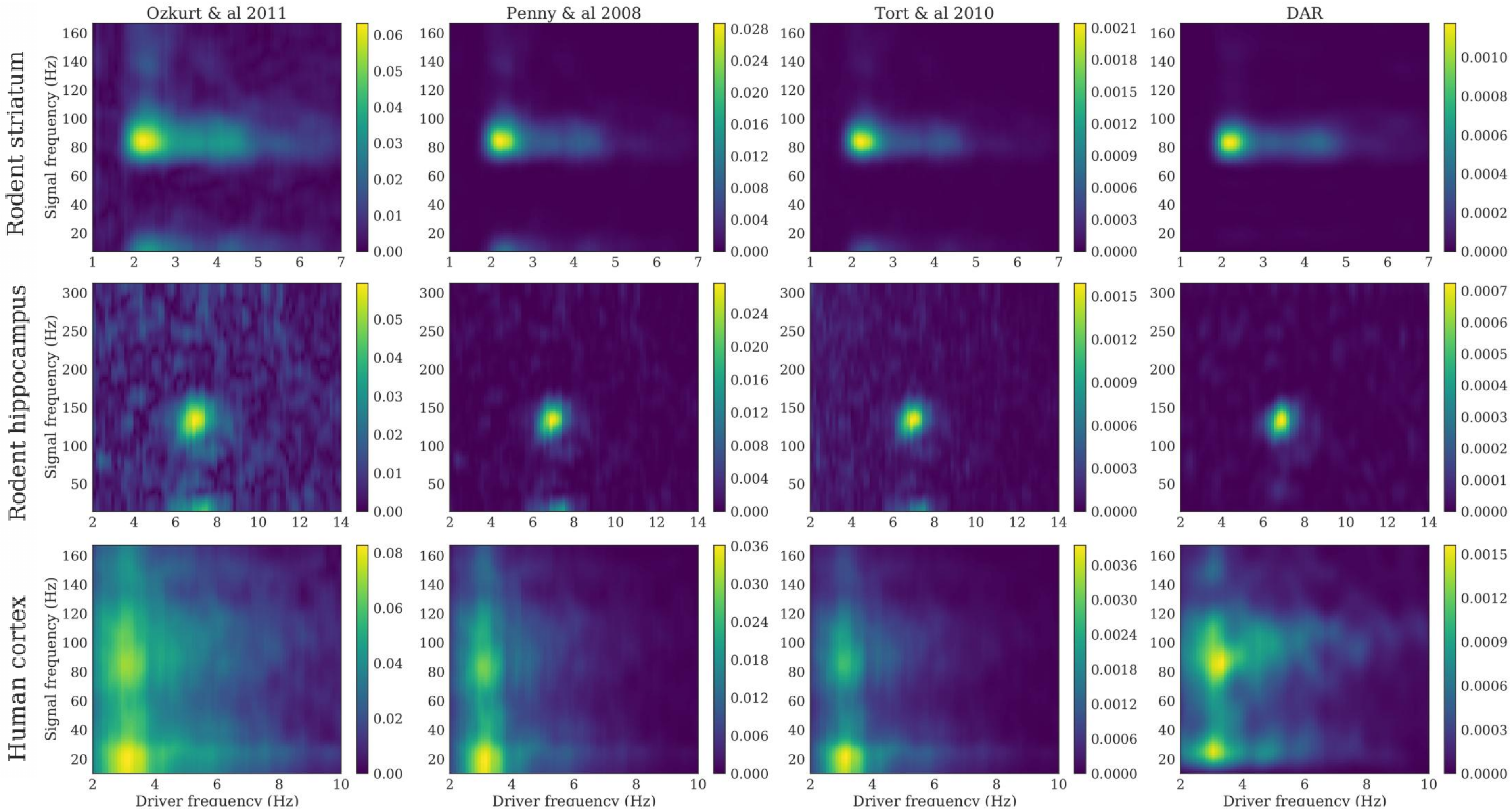


Comodulogram on 3 empirical signals



Model and parameter selection

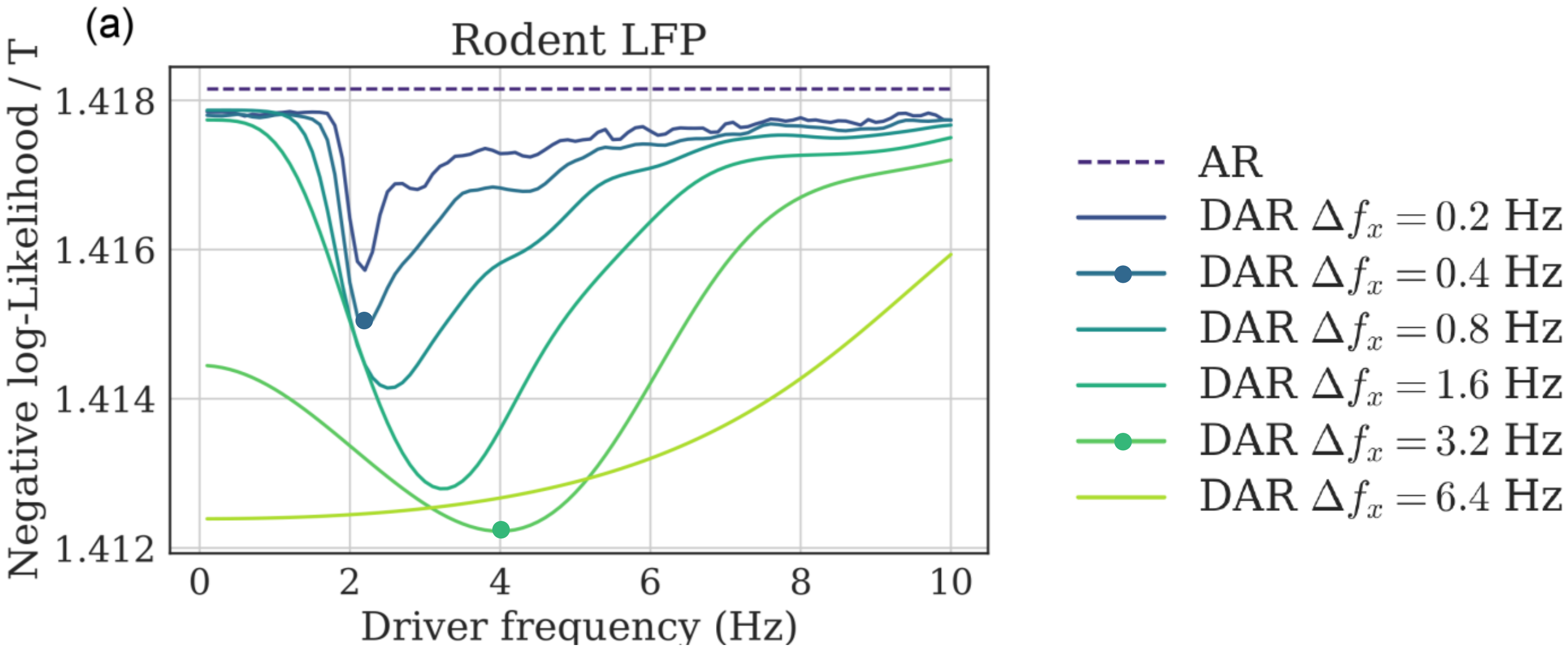
- Likelihood function

$$L = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{\varepsilon(t)^2}{2\sigma(t)^2}\right)$$

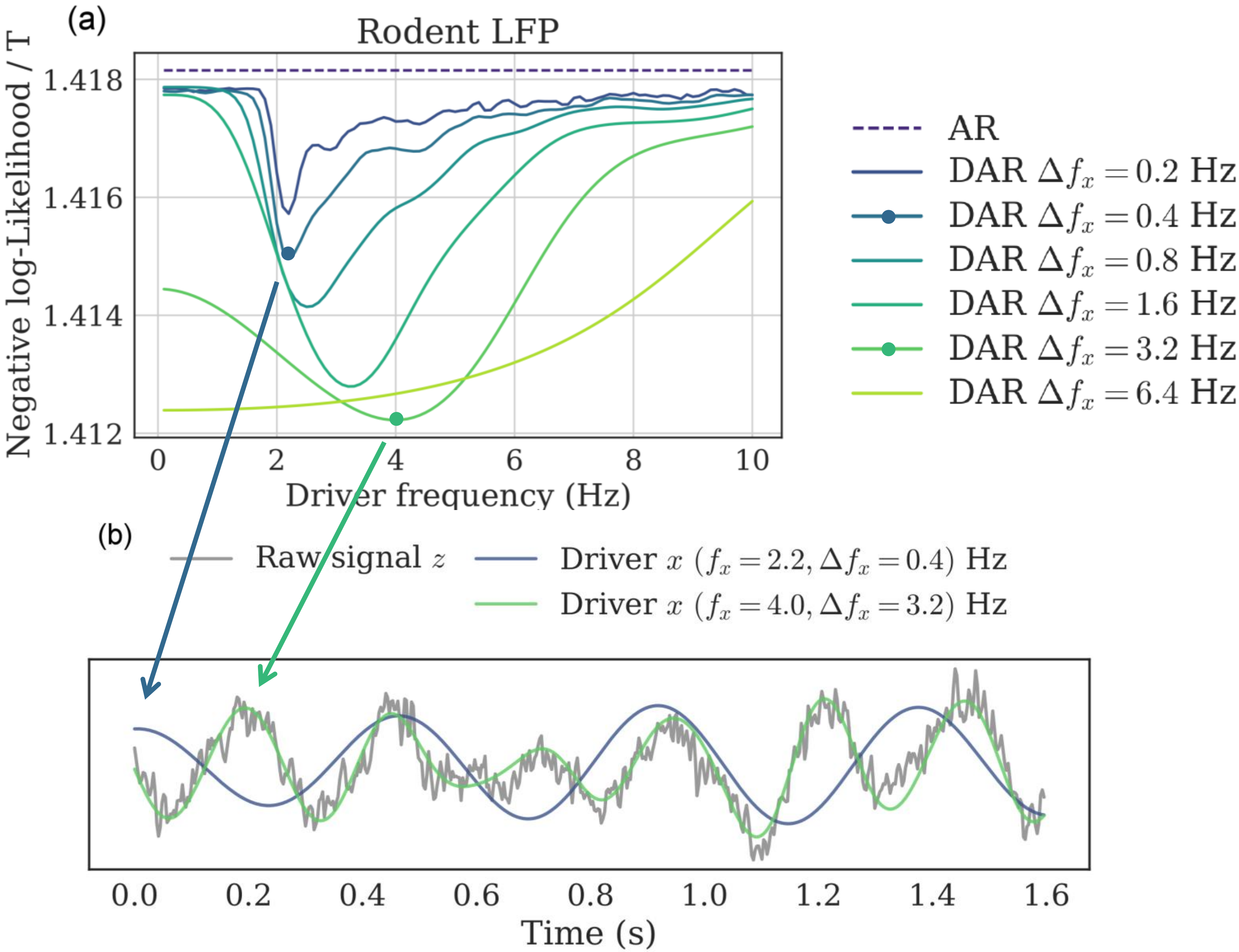
$$-2\log(L) = T\log(2\pi) + \sum_{t=p+1}^{T} \frac{\varepsilon(t)^2}{\sigma(t)^2} + 2\sum_{t=p+1}^{T} \log(\sigma(t))$$

→ Parameter selection

Driver selection



Driver selection



Delay estimation

- DAR model between $y(t)$ and $x_\tau(t) = x(t - \tau)$

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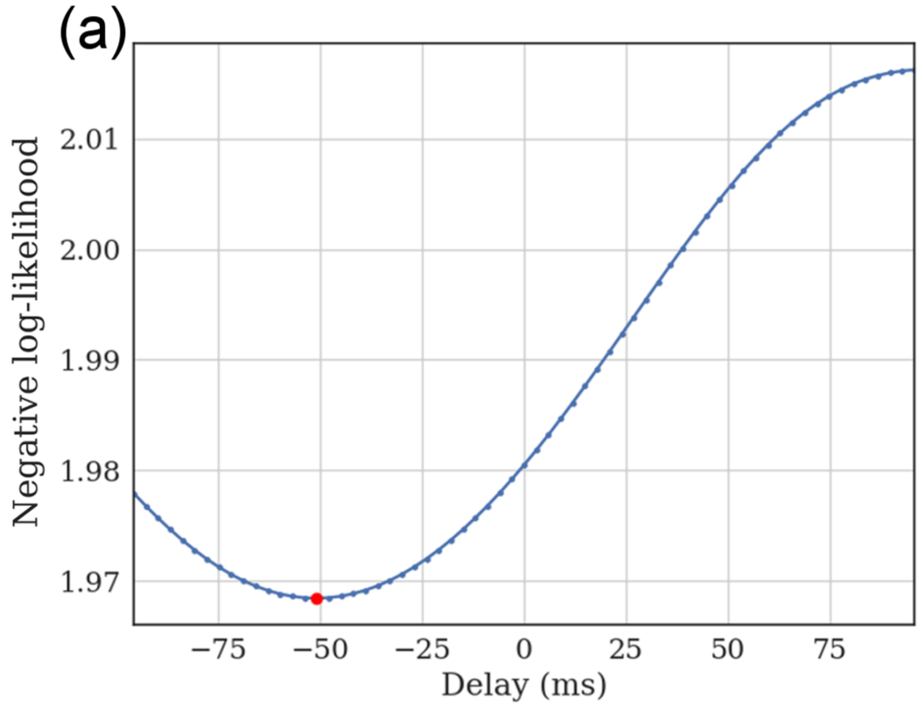
- Minimize the negative log-likelihood
- Add from direct and reverted time direction, to remove biases



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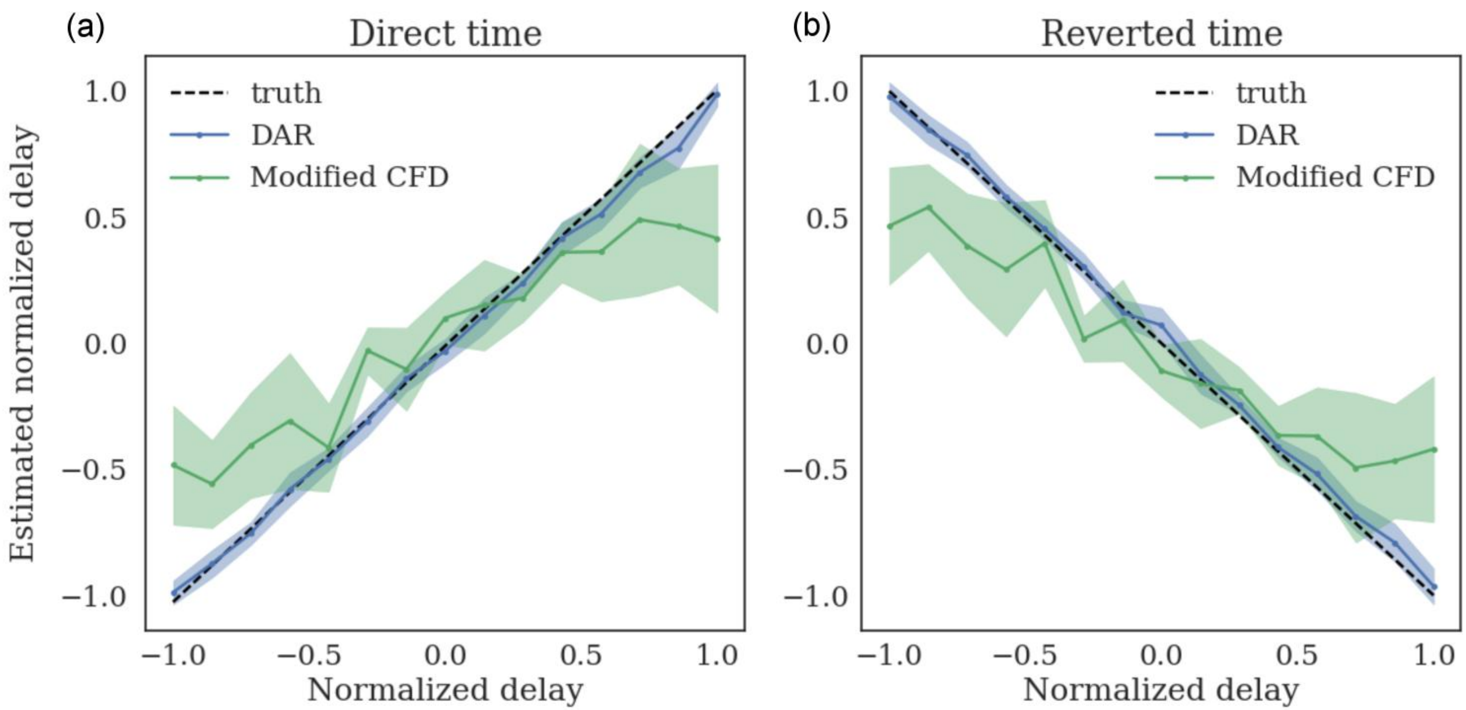
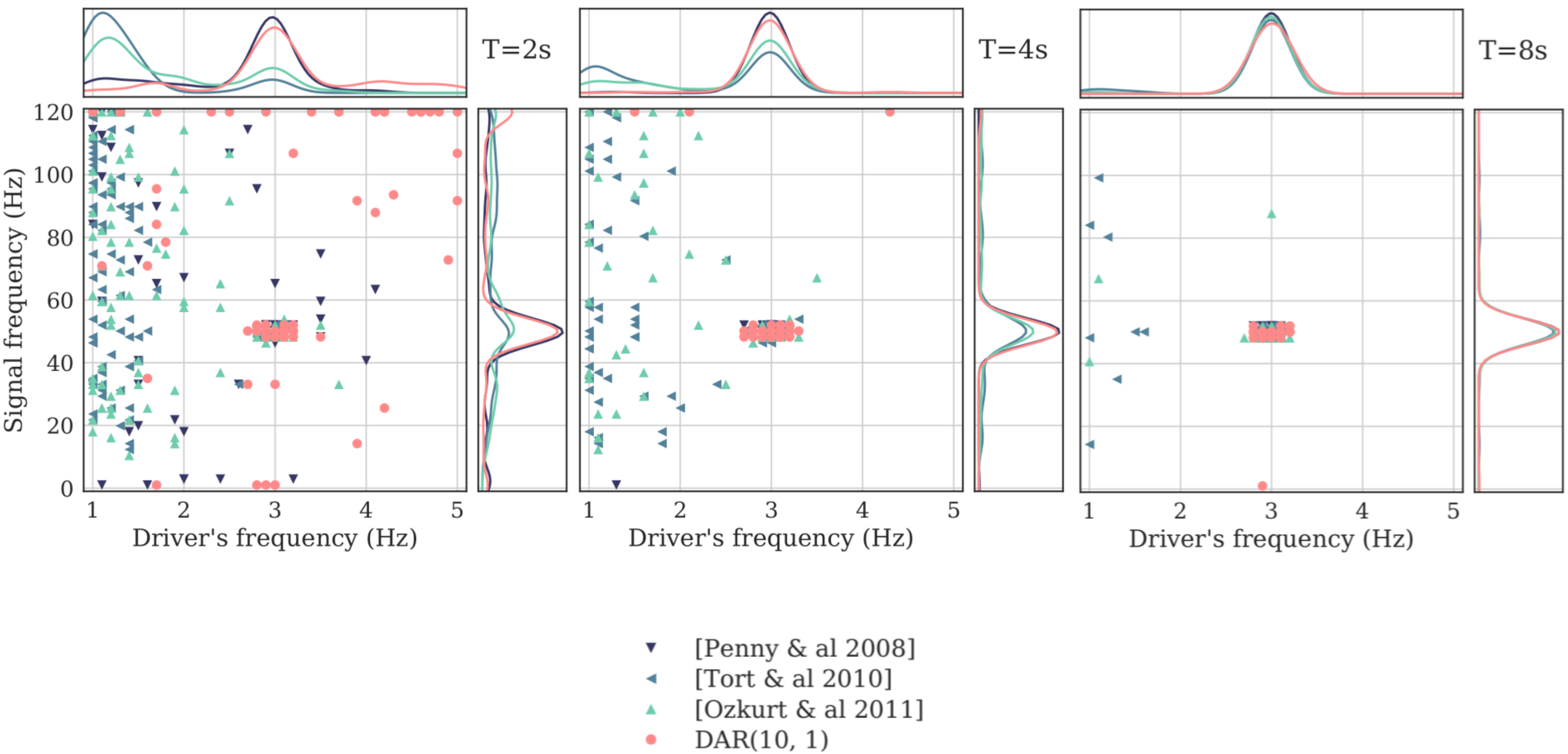


Robustness to short signals



Conclusion

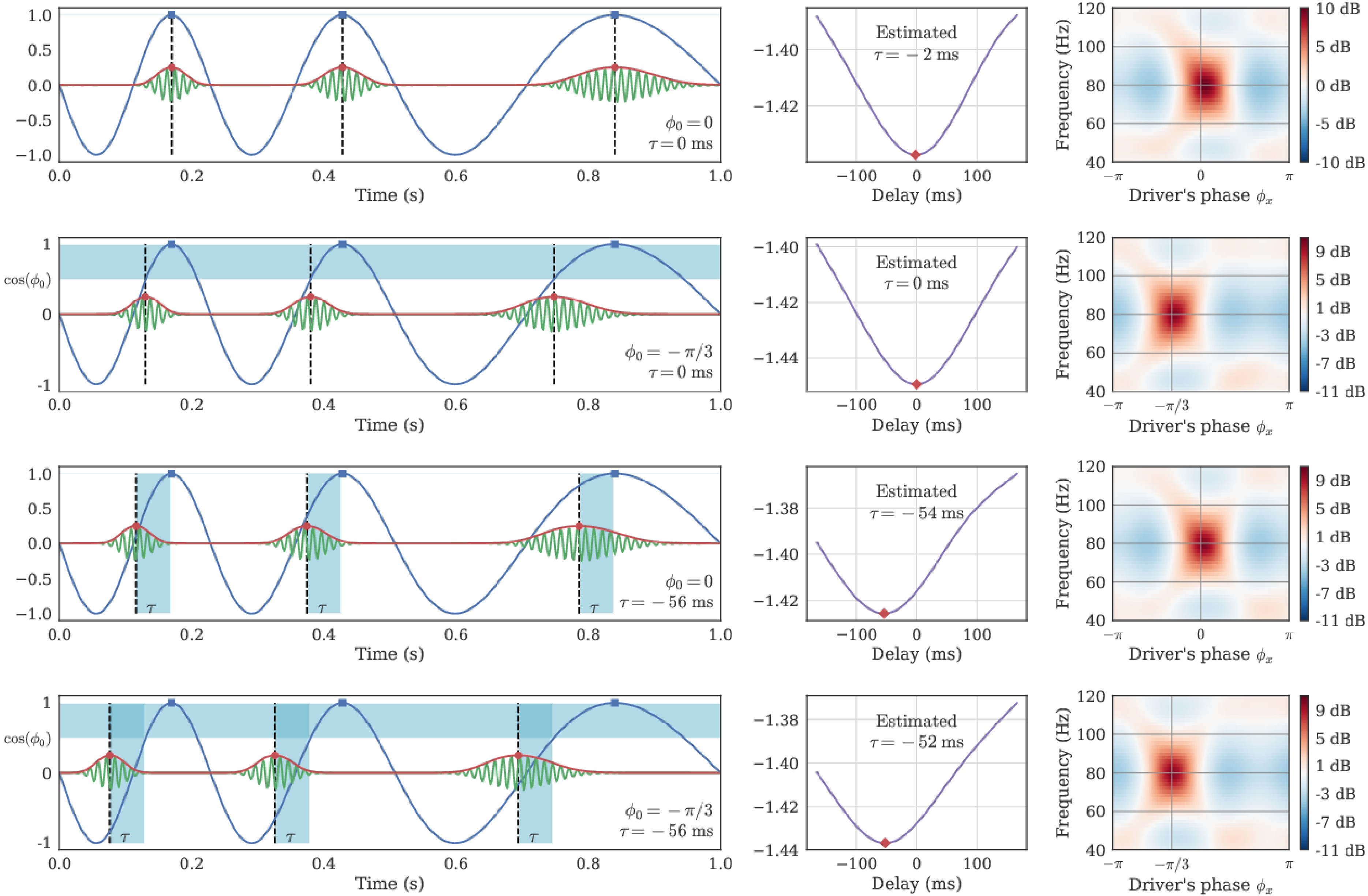
- Many PAC metrics, but
  - Pitfalls during filtering
  - No comparison possible (metrics or parameters)

- Driven Auto-Regressive (DAR) models
  - Estimation of spectral variation → capture PAC
  - Not affected by filtering pitfalls
  - Generative model → easy comparison of models/parameters
  - Delay estimation → directionality of the coupling
  - Parametric model → robust to short signals

Non-linear auto-regressive models for cross-frequency coupling in neural time series, Tom Dupré la Tour, Lucille Tallot, Laetitia Grabot, Valérie Doyère, Virginie van Wassenhove, Yves Grenier, Alexandre Gramfort, *bioRxiv preprint 2017*

https://github.com/pactools/pactools

Time delay and preferred phase



Guarantee local stability

- AR model

$$y(t) + \sum_{i=1}^{p} a_i y(t-i) = \varepsilon(t)$$

- Non-stationary AR model

$$a_i(t) = \sum_{j=0}^{m} a_{ij} x(t)^j$$

- Lattice parameterization

$$a_p^{(p)} = k_p; \qquad \forall i \in [1, p-1],\ a_i^{(p)} = a_i^{(p-1)} + k_p a_{p-i}^{(p-1)}$$

- Local stability criterion

$$-1 < k_i < 1$$

- Log Area Ratio

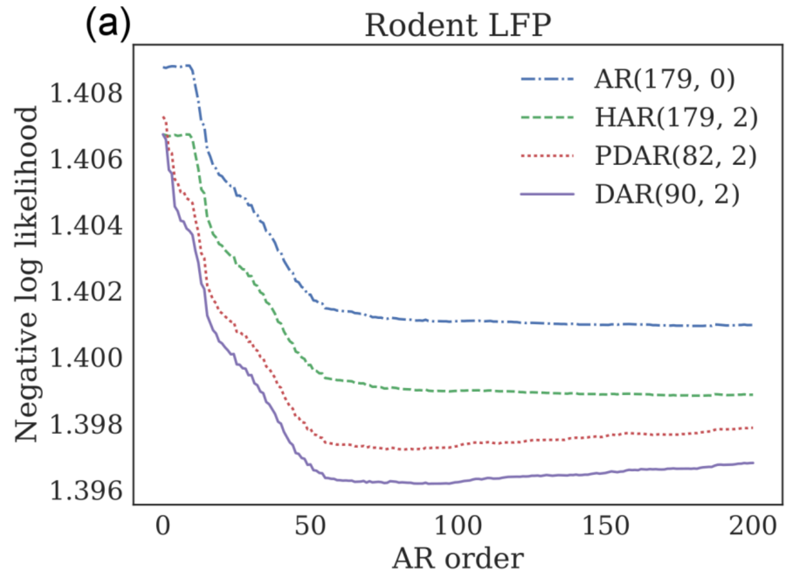
$$\gamma_i = \log\left(\frac{1+k_i}{1-k_i}\right) \iff k_i = \frac{e^{\gamma_i}-1}{e^{\gamma_i}+1}$$

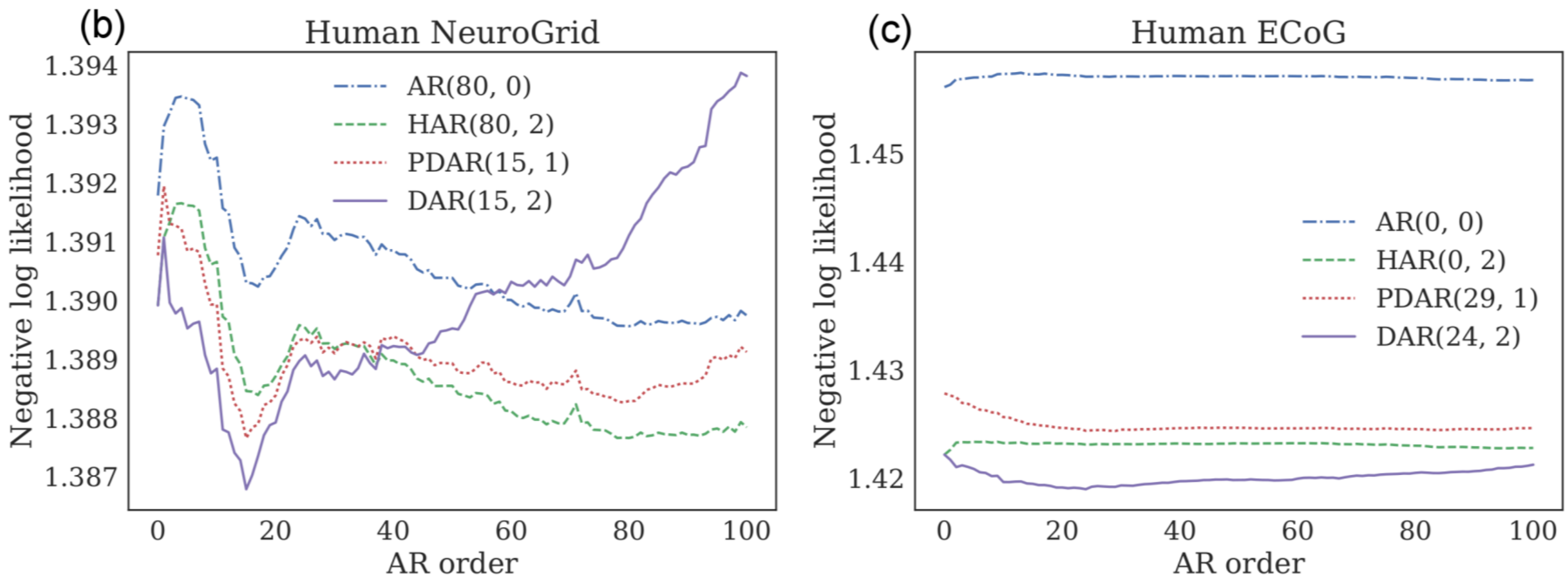
- Driven AR model

$$\gamma_i(t) = \sum_{j=0}^{m} \gamma_{ij} x(t)^j$$

Model variants and cross-validation

- AR: linear AR model
- HAR: linear AR model + driven innovation variance
- PDAR: driven AR model with constant amplitude driver
- DAR: driven AR model





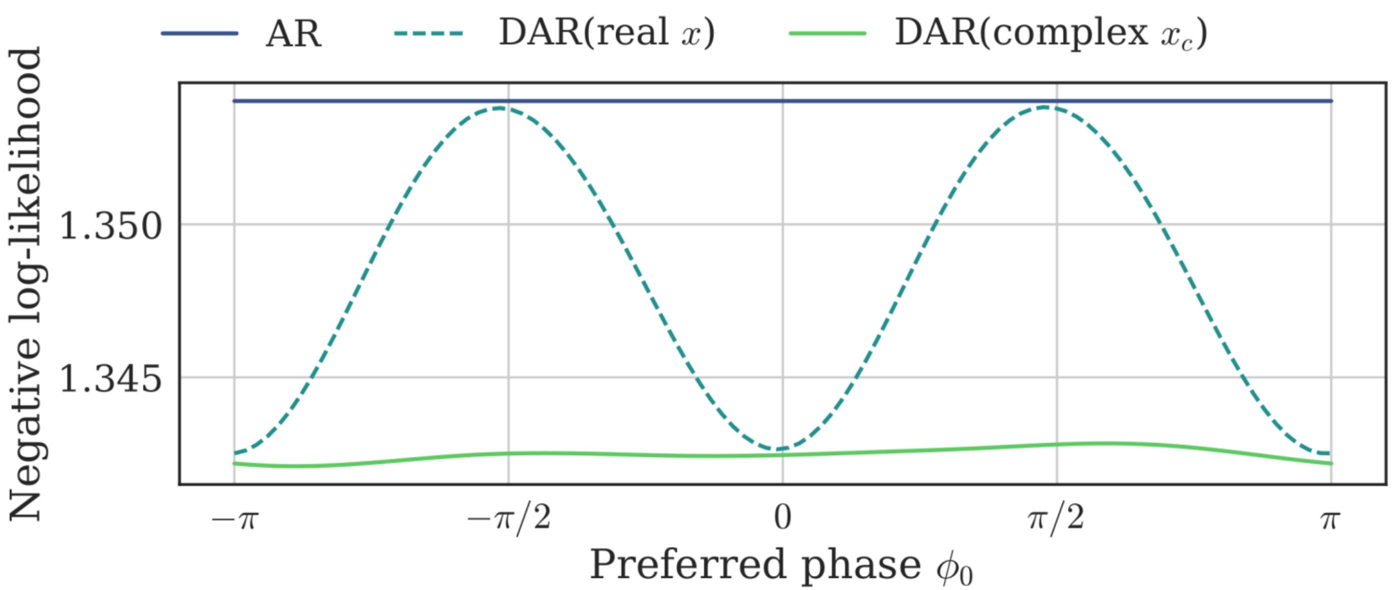
Using a complex driver

- With a real driver

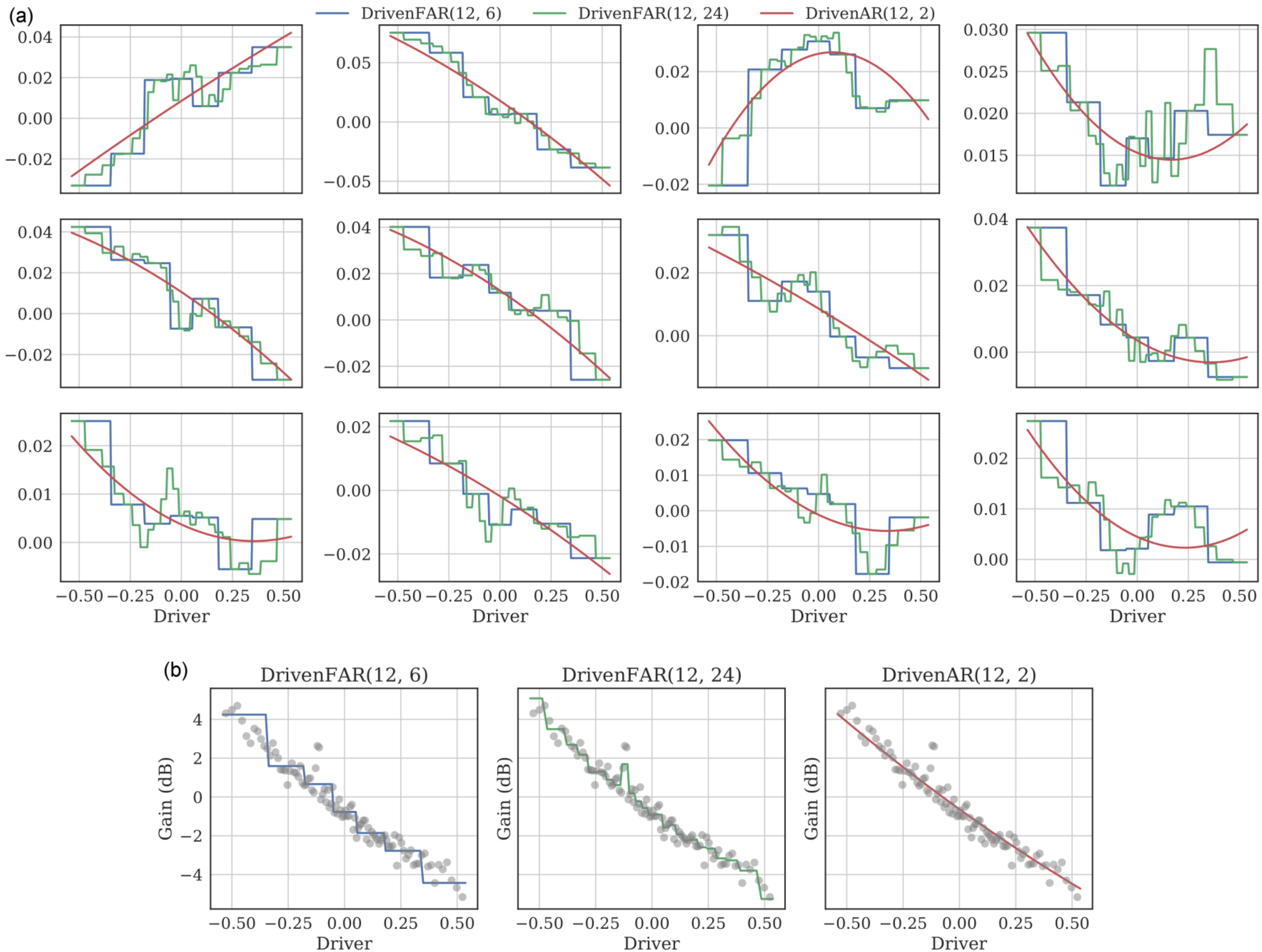
$$a_i(t) = \sum_{k=0}^{m} a_{ik} x(t)^k$$

- With a complex driver

$$a_i(t) = \sum_{0 \leq k+l \leq m} a_{ikl} \, x(t)^k \bar{x}(t)^l$$



The polynomial basis is good enough



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- BIC selection

$$BIC = -2\log(L) + d\log(T)$$
$$d = (p+1)(m+1)$$

- Testing the limits